Gambling for Redemption or Ripoff, and the Impact of Superpriority*

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ABSTRACT

Asset substitution by firms is gambling implemented by switching to inefficient and risky projects. Gambling using derivatives is more precise, gambling only to what is needed, with negligible efficiency loss. Optimal gambling can be small-scale "Gambling for redemption," which is socially beneficial, or large-scale "gambling for ripoff," which is socially inefficient, benefiting firm owners at the expense of bondholders. Gambling at scale is available with weak property rights, or in the U.S. through Qualified Financial Contracts (QFCs) with "superpriority" exemptions from bankruptcy provisions. Availability of gambling at scale reduces firm borrowing and value due to anticipation of gambling for ripoff.

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1 Introduction

In the early days of Federal Express, the company's cash once dwindled to \$5,000, too little to cover the \$24,000 jet fuel bill due the following Monday. With the firm hanging on the edge, the founder Frederick Smith flew to Las Vegas over the weekend and played blackjack to convert the \$5,000 into \$32,000, enough to keep the company afloat for another week.[1] While this gamble was obviously beneficial to the firm's owners because it provided a positive probability of avoiding bankruptcy, it was probably also beneficial for other claimants, including the fuel company, which would have received little in bankruptcy. Gambling by a firm can also benefit owners at the expense of creditors, as in asset substitution,[2] in which the upside benefit of gambling is received by the owners, and the downside is borne by creditors. In this paper, we study pure gambling by a firm using derivatives that allow more control over the payoff distribution and negligible inefficiency of investment compared to asset substitution. We can understand the impact of gambling through two polar cases in a single-period model. Gambling for redemption, as exemplified by the Federal Express scenario, involves gambling just enough to stay in business. Such gambling is good for the owners, the creditors, and for overall efficiency, because it minimizes the expected loss of continuation value. Gambling for ripoff, which operates at a larger scale, benefits the owners at the expense of the creditors and overall economic efficiency because it maximizes the expected loss of continuation value.

In the US, gambling for ripoff is of special interest because of controversial legislation before the 2008 financial crisis that exempts qualified financial contracts (QFCs), such as repos and financial derivatives, from important provisions of bankruptcy law, including automatic stay and clawbacks from preferential treatment. These contracts are referred to as

[1] Frock (2006)

[2] Early papers related to asset substitution include Black and Scholes (1973), Merton (1974), Jensen and Meckling (1976), Galai and Masulis (1976), and Myers (1977). Only Galai and Masulis (1976) seems to use the term "asset substitution."

superpriority claims[3] by Roe (2010), who asserts that they accelerated the financial crisis.[4] Superpriority laws make it easier for firms to gamble with their assets, allowing for gambling at a larger scale and in the presence of accounting controls not present in the FedEx example. In subsection 1.1 we will discuss superpriority laws in detail. For now, it suffices to note that when applicable, superpriority laws increase the scale of available gambling, and the increased scale makes gambling for ripoff more appealing to the owners. In subsection 1.2, we discuss similar effects that arise from weak bankruptcy laws and poor specification or enforcement of property rights in underdeveloped regions.

We start with a single-period model in which superprioprity laws are necessarily good for firm owners, because it gives them more flexibility, and critically because the amount of debt and the continuation value are both exogenous. In the subsequent multi-period model, potential future gambling is reflected in bond pricing, and bond investors understand that superpriority increases the likelihood of gambling for ripoff. As a result, bond investors demand higher returns to compensate for the increased risk, leading to a decrease in what can be borrowed and a decrease in the firm's overall value.

Gambling in our model does more than merely shift value from bondholders to owners, a zero-sum game in looting models of Akerlof et al. (1993) and Boyd and Hakenes (2014). It also potentially mitigates or exacerbates the social loss of the firm's continuation value. When a firm defaults, its continuation value is typically lost. But through gambling, the firm might preserve this value with some probability of winning.[5] Conversely, if a firm is in a good position to repay its debt and maintain operation, gambling can introduce a risk of

[3]Priority is the promised order of satisfaction of claims in bankruptcy. The QFCs technically do not have a priority, since they are exempt from bankruptcy law, but in effect their exemption is like having higher priority than all other claims, hence the term "superpriority."

[4]Roe (2010) argues that these laws undermined creditors' incentives to monitor the firm and creating a too-big-to-fail problem. However, the implementation of superpriority laws has its boundaries. Notably, when FDIC-insured depository institutions fail, they are resolved under the FDIC, and currently their Qualified Financial Contracts (QFCs) are not exempt from the automatic stay. More discussion regarding the scope of the law can be found in subsection 1.1.

[5]With additional loss incurred during bankruptcy proceedings, such as if we take into account bankruptcy costs for bondholders, gambling could further offset the loss to an extent by providing a chance of avoiding bankruptcy.

failing and forfeits the continuation value with some probability, which is a social loss.

Let's consider a simple example of gambling for redemption or ripoff. Assume $C \geq 0$ is the continuation value that will be lost in bankruptcy, and for simplicity, the example assumes the liquidation value of these assets is 0 and it is not possible to issue new debt.[6] In the absence of gambling, the equity value can be expressed as $(\pi \geq F)(\pi + C - F)$, where $\pi > 0$ is available cash and $F > 0$ is the face value of debt due now, while the bond value is given by $\min\{\pi, F\}$. Similarly consider a gamble that pays $P \geq 0$ with a probability of $\frac{\pi}{\pi+P}$ and $-\pi$ with a probability of $\frac{P}{\pi+P}$. We will assume that $\pi + P \geq F$, as otherwise the equity value would always be 0. When the gamble loses, paying bet $-\pi$, the firm has no assets, but $\pi + P \geq F$ implies that the equity is in the money in the good state when the gamble pays P. The expected equity value with this gamble is

$$\frac{\pi}{\pi+P}(\pi + P + C - F) = \pi + \frac{\pi}{\pi+P}(C - F),$$

where $C - F$ is the net gain from continuation. If this net gain of continuing, $C - F$, is negative (as it is if $C = 0$), it is optimal for the equity holders to choose the largest possible value for P. In the limit, as P increases, the equity holders obtain expected value π, while the bondholders receive nothing. This is "gambling for ripoff." However, if the net gain of continuing, $C - F$, is positive, it is optimal to choose $P = F - \pi$ if $\pi < F$. If $\pi \geq F$ and $C - F > 0$, not gambling is optimal and so is gambling at small enough scale so π plus gambling losses always covers F. When there is a cash shortfall ($\pi < F$), the owners would "gamble for redemption," choosing $P = F - \pi$, so that the cash after gambling ($\pi + P$) exactly covers the debt (F). In this case, the equity value would be $\frac{\pi}{F}C$, where $\frac{\pi}{F}$ is the probability of the firm continuing. The bondholders obtain

$$\frac{\pi}{F}\min\{F, F\} + \frac{F - \pi}{F}\min\{\pi - \pi, F\} = \pi,$$

[6]With positive liquidation value and new debt, the problem becomes more complicated. Our single-period model in Section 2 introduces these variables, which are endogenized in the multi-period model in Section 3.

which means they are indifferent about gambling for redemption, and bankruptcy costs would imply they are better off with gambling.

It may seem strange that bondholders are indifferent about gambling for redemption, given that with risk-neutral probabilities in a single-period model, their payoff is concave in firm value, so Jensen's inequality implies that gambling should make them worse off.[7] This argument is the basis of equity's incentive for traditional asset substitution. However, gambling with derivatives is more precise. When the owners gamble for redemption using derivatives, they design the gamble to yield an amount precisely equal to the required debt payment (for example, using a digital option), so that the gamble is confined to the left tail of the distribution where the bond payoff is linear. Consequently, bondholders are indifferent about gambling for redemption, and in fact, they can even benefit if there are bankruptcy costs. But traditional asset substitution involves gambling that surpasses the required payment threshold, which corresponds to the point of concavity in the bond's payoff distribution. As a result, bondholders are worse off. This is shown in the previous literature which typically used normal or lognormal distributions for risk, so that taking on more risk increases the probability of payoffs on both tails.[8] Additionally, traditional asset substitution can be inefficient as it often entails firms undertaking wasteful projects. These projects tend to have extended timelines for returns and have unwanted noise and informational ambiguities. But derivatives markets are more cost-effective, and short-term gambling can minimize the arrival of confounding information so it is clear exactly how much to gamble.

The single-period model suggests that gambling is not a problem in normal times when it is beneficial for the firm owners to keep the firm running. In these times, the owners have no incentive to choose risky gambles that could lead to default, and even if the firm experiences temporary negative shocks on cash flow, the owners prefer beneficial gambling

[7] Jagannathan (1984) shows that adding noise in the risk-neutral probabilities increases (resp. decreases) the value of a convex (resp. concave) claim. Since we assume risk-neutral pricing, actual and risk-neutral probabilities are the same in our model.

[8] Ericsson (1997) studies firm's one-time choice of risk which is either at a high level or at a low level. Gong (2004), Ross et al. (1998), Leland (1998) and Della Seta et al. (2020) extend the choice of variance to an interval.

for redemption. Gambling becomes a problem in bad times when the firm's continuation value is small compared to its debt, presumably due to some bad luck. In such cases, the owners would prefer gambling for ripoff, as it maximizes their benefits by looting the value that should have gone to the bondholders. Interestingly, in this case the owners favor such extreme gambling regardless having enough cash to cover debt or not. When cash is insufficient, gambling for ripoff transfers value from the bondholders to the owners. But if there is enough cash to cover the debt, gambling for ripoff also dissipates the equity's continuation value. In this scenario, providing liquidity to save the firm may help keep it running temporarily, but it may not be sufficient to change the risk-taking behavior of the owners. Instead, policies or institutions that increase the firm's continuation value or prevent large-scale gambling may be more socially efficient.

One possible weakness of the single-period model is exogeneity of the face value of maturing debt. This might be a good assumption at the time of the superpriority legislation, if the legislation is a surprise to the bondholders with existing debt. However, to understand the impact of the law once it is understood by bondholders and priced into the debt, a multi-period model with endogenous debt is more useful. In each period of the multi-period model, the owners choose gambling and new financing after a capital shock is realized. In the multi-period model, the optimal gambling strategy may not always be a polar case of gambling for redemption or gambling for ripoff, but may instead lie somewhere in between. When the continuation value is sufficiently large, gambling tilts towards gambling for redemption, and when the continuation value is sufficiently small, there is gambling for ripoff. Intuitively, the benefit of gambling depends on the frequency of gambling tilting towards gambling for redemption versus ripoff. Our main result of the multi-period model shows that if there is significant liquidation value to gamble with (for example due to superpriority), gambling reduces the maximum amount the owners can borrow, and also reduces the market value of equity. This suggests that superpriority can benefit owners of firms sufficiently far underwater if it is a surprise at the time of passage, but may not benefit owners once lenders understand that the law can make large gambles more attractive to owners.

If the firm owners are potentially worse off due to superpriority laws, as suggested in our multi-period model, they may resort to more defensive measures (operating leverage, secured debt, short-term debt, and even repos) to protect against the laws. Furthermore, negative pledge covenants in bankruptcy may no longer offer sufficient protection for bondholders, and they may need to rely more on perfected security interests under UCC Article 9. This is supported by some empirical evidence. For example, Benmelech et al. (2020) Figure 8a documents an increase of secured debt over total debt since 1995 and an upward jump in 2005. Baily et al. (2008) Figure 6 shows that the issuance of total value of short term (with 1-4 days maturity) asset-backed commercial paper has increased significantly from 2005 to mid 2007, whereas the commercial paper with longer terms (with 21-40 days and > 40 days maturities) stayed steady during the period. There was also a surge in the growth in the market for repurchase agreements, a much higher growth rate compared to the total debt in the financial sector, particularly after 1999 (Roe (2010)). Ganduri (2016) finds a surge of the number of repurchase agreements after BAPCPA went into effect in 2005, whereas the number of loans plummeted during the same period; Lewis (2020) provides causal evidence of expansion of repo collateral rehypothecation as a result of the law and estimates a money multiplier of private-label mortgage collateral to be 4.5 times that of Treasuries. However, relying on perfected collateral to prevent large gambles carries its own costs, including the inflexibility of assets redeployment and constraints on future borrowing and investment. This has been discussed in Donaldson et al. (2019, 2020).[9] Our model is consistent with defensive measures, which would impact the parameter that gives the fraction of capital that can be gambled through superpriority.

The paper is structured as follows. The following subsection 1.1 provides more details on superpriority laws and the implication of gambling in the developing economy. Section 2 focuses on the two polar cases of “gambling for redemption“ and “gambling for ripoff”

[9] In turn, these costs can be mitigated somewhat by issuing collateralized debt with a call provision or a short maturity.

by examining a stripped-down single-period model, and with discussions of other potential applications of our single-period model. Section 3 develops a multi-period model using the building block in Section 2 and incorporates endogenous decision making to study the ex-ante effects of gambling with and without superpriority. Section 4 characterizes equilibrium properties of the model and provides numerical examples to illustrate the results, and Section 5 concludes.

1.1 Superpriority of QFCs and gambling

Traditionally, redeploying assets to compensate claimants with amounts exceeding what they would have received through bankruptcy proceedings has been difficult. While common law permits asset seizure to satisfy debts, seizure or sales can be clawed back in bankruptcy. Specifically, if an asset transfer occurs within 90 days (or longer period in some instances)[10] of the filing of bankruptcy, it is considered a preferential treatment if the firm is unable to equally satisfy all the creditors, and the transfer is subject to reversal by the court. The purpose of these provisions is to prevent a frenzied competition among creditors to grab assets in the firm, and to provide fair treatment of all the claimants. In addition, bond covenants can be used to trigger bankruptcy in response to asset seizure or sales. Such covenants often contain clauses that limit the firm's ability to sell its assets, typically placing the firm in default on its loans if the covenants are violated. Moreover, cross-default clauses in bond agreements which stipulate that a default on one bond triggers on all outstanding bonds, normally result in the firm entering bankruptcy. All these provisions tend to make asset seizure or sale pointless.

In other words, traditional institutions implied that any promise by the firm to transfer assets to cover losses from gambles would not be credible for the gambling counterparties unless they were sure that the firm will not be forced into bankruptcy. This lack of credibility constrains the scale of gambling.

[10]The clawback of preferential transfers extends back one year for a preferential transfer to an insider, or up to two years for a fraudulent conveyance.

However, for entities resolved under the US Code Title 11, Chapter 7, Chapter 11, and Chapter 15, the QFCs receive an exemption. This exemption sidesteps the usual legal protections for assets and instead prioritizes the rights of derivatives counterparties. Specifically, it allows them to "terminate, liquidate, or accelerate" derivatives contracts even before bankruptcy proceedings begin, thus provides "superpriority."[11] These superpriority rights provide a significant advantage to gambling counterparties, because these laws ensure that the assets can be collected without being stayed in the firm's estate during bankruptcy proceedings. Consequently, the firm and their gambling counterparties can make commitments to pledge the firm's assets in gambling, knowing that these commitments will remain unimpeded by bankruptcy law.

The "superpriority" claims we are talking about obtained their exemption from bankruptcy in a series of laws passed between 1978 and 2006. See Schwarcz and Sharon (2014) for a detailed history of the law. The game changer appears to have been the 2005 amendment to the bankruptcy code, known as the Bankruptcy Abuse Prevention and Consumer Protection Act (BAPCPA), which broadened the scope of the exemption to include all derivative securities. which started with some commodity futures and previously extended to repos and swaps, to all derivative securities. Taken together, these laws exempt qualified financial contracts (QFCs) - which encompass a wide range of financial instruments such as securities contracts, commondity contracts, forward contracts, repos, and swaps - with immunity from the automatic stay and clawbacks that typically arise in bankruptcy proceedings.[12] In addition, BAPCA and the subsequent 2006 Act introduced further protections for "master netting agreements" relating to the QFCs mentioned above. These agreements allow counterparties to offset mutual obligations. For instance, if two firms owed each other one dollar, without netting, when one firm is in bankruptcy the counterparty has to repay the one dollar

[11]See 11 U.S. Code §362(b)(6), §546(e).

[12]Supepriority also favors derivatives by exempting clawbacks of constructive (but not actual) fraudulent transfers. See Vasser (2005). Nonetheless, it is worth noting that the exemption may not be applicable in cases of gambling, where the transfer is made in satisfaction of a pre-existing claim and represents a fair value exchange, which probably cannot be defined as fraudulent.

and may receive only 50 cents out of the dollar from the firm. With a netting agreement, the counterparty can offset the debt and be paid 100 cents out of a dollar. This treatment makes gambling even easier.

The superpriority treatment has drawn a lot of attention since the 2008 financial crisis. Roe (2010) observes a soaring volume of interest rate derivatives from $13 trillion in 1994 to $430 trillion in 2009, representing almost a forty-fold increase. During the same period, private business debt only tripled from $11 trillion to $34 trillion.[13] Baily et al. (2008) also shows an exponential growth in the value of outstanding CDS since 2001. Roe argues that this is because superpriority provides a cheaper way of financing, facilitating greater liquidity that would not have occurred otherwise. This shift away from traditional financing also reduces the incentives of derivatives counterparties to monitor the firm, exacerbating the "too big to fail" problem if systemically important firms rely heavily on the these derivatives. In addition to the costs, Duffie and Skeel (2012) highlights the benefits of the safe harbor exemption on QFCs, such as ensuring the redemption of critical hedges and reducing self-fulfilling security runs, alhough it is also possible that superpriority causes runs to grab assets that were previously deterred by the automatic stay and clawbacks. Previous economic literature has focused on the fire sales in the repo market, which dilute the collateral value for the secured creditors.[14] Instead, our paper focuses specifically on gambling.

While Chapter 7 and Chapter 11 under Title 11 Bankruptcy Code apply to most individuals and entities in the United States, several parallel provisions govern the bankruptcy process of other entities. While these provisions borrow the basic concepts from the Bankruptcy Code, each is with their own specific rules for treating QFCs. For instance, the bankruptcy of FDIC-insured banks is managed by the FDIC under the Federal Deposit Insurance Act (FDIA), in which domestic QFCs are subject to a stay by the statutes which supersede the Bankruptcy Code. On the other hand, stockbrokers' liquidation is typically managed

[13] Roe (2010) Figure 1.

[14] See Infante (2013), Oehmke (2014), Antinolfi et al. (2015) and Auh et al. (2018).

by the Securities Investor Protection Corporation (SIPC) under SIPA.[15] Additionally, the bankruptcy processes for insurance companies are overseen by state law, with variations where some states have enacted statutes that parallel the FDIA or the Code (excluding QFC provisions), while others have adopted QFC provisions analogous to those in the FDIA. However, if an entity is considered as a systemically important financial company, its bankruptcy procedure is managed by the Orderly Liquidation Authority (OLA) under Title II of the Dodd-Frank Act, which takes precedence over any bankruptcy filings. Contrary to procedures under the Bankruptcy Code, QFCs experience a 24-hour stay under OLA, during which the FDIC selects whether to transfer them to a bridge company or a third party.[16] It is unclear to us to what extent this transfer will block QFC counterparties from collecting on the contracts.

1.2 Gambling and underdeveloped legal systems

Besides superpriority, it is possible to consider broader implications of gambling on the deficiencies in the legal systems of underdeveloped countries, which also allows gambling away assets. As highlighted by Soto (2001), a significant proportion of assets in countries such as Peru, Egypt, Philippines, and Haiti are held by the poor. Due to the prohibitively expensive and complex legal procedures of documenting ownership, these assets often lack registration and protection under the law.

When individuals and businesses encounter financial difficulties and are unable to meet their debt obligations, they can resort to a short-term gambling, by promising their gambling counterparties higher priority to seize assets in case of failure. When they lose, their gambling counterparties could seize the houses and vehicles immediately without any legal repercussions. Consequently, our single-period model suggests that, all else being equal, there would be stronger temptation to engage in gambling for ripoff in comparison to scenarios where the legal system provides protection for these assets. Besides enabling individuals

[15] Refer to 15 U.S. Code §78eee(b)(2)(C).

[16] For more details, see Gottlieb (2017) and Treasury (2018).

to gamble away assets, this inclination is also partly due to the low collateral value of the assets.

2 Optimal gambling: the single-period model

We start with a stripped-down model to identify two different types of gambling: gambling for redemption and gambling for ripoff. Gambling for redemption occurs when the firm cannot immediately pay off its debt, and bankruptcy would result in a net loss for the owners. In this case, the owners will only take necessary risks to meet their debt obligations and avoid bankruptcy, benefitting both the owners and the bondholders. In contrast, gambling for ripoff occurs when the owners would gain in net from bankruptcy. In this case, owners will gamble to a large payoff to maximize the probability of bankruptcy, allowing them to evade debt obligations while still collecting the firm's unprotected liquidation value, benefiting the owners at the expense of the bondholders. Superpriority claims can reduce the net loss to owners by enabling them to directly collect a portion of the firm's asset value without having to pay. This result in bigger gambles.

We are looking at a single-period model intended to mimic a snapshot at a node where a firm has debt of $\boldsymbol{F} > 0$ maturing now. The crucial outcome is whether this firm, if socially valuable, continues or not. If the debt cannot be fully repaid, the firm undergoes a liquidation (as in Chapter 7 bankruptcy), and all assets are sold to repay the debt. In this section, we assume that $\boldsymbol{F}$ is positive so that bankruptcy is possible. In the multi-period model, debt can be negative, representing savings, but that is not relevant for this section's purpose. Bankruptcy has costs, as owners receive nothing, and therefore they typically prefer to pay off the debt and continue. The owners can repay the debt using a combination of (1) cash flow $\boldsymbol{\pi} \geq 0$ generated from operations, (2) proceeds from rolling over some of the debt, and (3) net gambling proceeds $\boldsymbol{G}$. We assume that the maximum debt that can be rolled over, denoted by $\boldsymbol{B}$, is smaller than the total debt amount $\boldsymbol{F}$, or else gambling would not be necessary. The owners are only allowed to choose gambles with negative payoffs that can be credibly repaid. We implement this requirement as a constraint on the owners' optimization problem, which depends on whether gambling is available and on the fraction of liquidation value that can be used to settle superpriority claims.

We show the economic balance sheets of the firm in Table 1 given a gambling payoff G. Balance sheet (1) is a general sheet, with some values left blank and to be calculated based on whether the firm continues or not. If the sum of cash flow π and net gambling outcome G is greater than or equal to the debt amount F minus the maximum debt that can be rolled over B, i.e., $\pi + G \geq F - B$, then the firm can continue. In this case, the entire value of the firm before raising any new funds includes the continuation value C, cash flow π and net outcome from gambling G. The existing bond has a value of F, and the remaining value, $C + \pi + G - F$, goes to the owners. However, if the sum of cash $\pi + G$ cannot cover the required amount $F - B$, all bondholders and owners bear a fractional bankruptcy cost $c \in [0, 1]$ of their respective values. The owners always receive nothing, and bondholders receive the residual value, which is equal to the liquidation value plus any cash after paying the bankruptcy costs, i.e., $(1 - c)(L + \pi + G)$.

In balance sheet (3), we also make the following assumptions to focus on the interesting cases:

Assumption 1 $C \geq B \geq L \geq 0$.

In this assumption, $C \geq B$ ensures that the maximum amount of debt can be rolled over (new liability) should not be greater than the continuation value of the firm, otherwise the owners would simply collect the cash, abandon the firm and walk away, making it pointless to discuss whether the firm will continue or not. The assumption of $B \geq L$ focuses our analysis on the interesting case because otherwise the owners could probably borrow for at least a little while to defer part of the current payment $F - B$ if $B < L$. (We will discuss more about this assumption at the end of this section.) $C \geq B$ and $B \geq L$ implying that $C \geq L$ and indicates that it is always socially efficient to continue the firm. Under these assumptions, we show the balance sheets in economic value after the realization of gambling payoff G.

(1)
general

Cash	$\pi+G$	Old bonds	? (face F)
Continuation value	?	Lawyers and accountants	?
Liquidation value	?	Equity	?

continues $\pi+G \geq F-B$

fails $\pi+G < F-B$

(2)
can pay off F

Cash	$\pi+G$	Old bonds	F
Continuation value	C	Lawyers and accountants	0
Liquidation value	0	Equity	$C+\pi+G-F$

(3)
cannot pay off F

Cash	$\pi+G$	Old bonds	$(\pi+G+L)(1-c)$
Continuation value	0	Lawyers and accountants	$(\pi+G+L)c$
Liquidation value	L	Equity	0

Table 1: ECONOMIC BALANCE SHEETS
The balance sheets are given in economic value, which is at a snapshot when a gambling payoff G is realized and before raising funds B from new bondholders or paying old bondholders the face value F. Balance sheet (1) is the general sheet, and the missing values filled with question marks are to be calculated based on whether the firm continues or not. For example, the existing bonds have a face value F, but the bondholders receive F only if the firm continues. Balance sheet (2) shows the corresponding values when the firm continues. That is, when $\pi+G \geq F-B$, the total cash can cover the required net payment. Balance sheet (3) shows the corresponding values when the firm fails. That is, when $\pi+G < F-B$, the total cash cannot cover the required net payment. The owners always receive zero, and bondholders receive the residual value, which is the liquidation value plus any cash, net of bankruptcy cost, $(1-c)(L+\pi+G)$.

The only uncertainty or endogeneity in the model is the gambling, i.e., F, π, B, C, c, L, and γ are all constants known to the agents. Taking all these variables to be constant is a better approximation than it may seem. At the time of a very-short-maturity gamble, the agents may not know what would come from a full liquidation of the assets, but they may know how much they would raise for an pre-arranged as-is sale (perhaps to the counterparty) at this point of time. The owners choose a gambling payoff $\mathbf{G}(\tilde{x})$ to maximize the expected equity value

$$\mathrm{E}[\left(\pi + B + \mathbf{G}(\tilde{x}) \geq F\right)(C + \pi + \mathbf{G}(\tilde{x}) - F)]. \tag{1}$$

All agents in our model are risk neutral, and $\mathrm{E}[\cdot]$ in (1) indicates either the common beliefs or risk-neutral expectations given agents' shared valuations.[17] We assume that $\tilde{x} \sim_d U(0,1)$ is the underlying randomness for gambling. The gambling function $\mathbf{G}$ maps the support of randomness to a set of the feasible gambling outcomes $\mathscr{O}$. Formally, the feasible gambling set is

$$\mathscr{G} \equiv \left\{ \text{non-increasing } \mathbf{G} : [0,1] \to \mathscr{O} \middle| \, \mathrm{E}[\mathbf{G}(\tilde{x})] = 0 \right\} \tag{2}$$

given the feasible gambling outcomes

$$\mathscr{O} = \begin{cases} \{0\}, & \text{no gambling} \\ [-\gamma L - \pi, \bar{G}], & \text{otherwise.} \end{cases} \tag{3}$$

To ensure a unique solution, the feasible gambling set in (2) requires $\mathbf{G}$ to be non-increasing. The fair gambling requirement, $\mathrm{E}[\mathbf{G}(\tilde{x})] = 0$, assumes negligible transaction costs, which is a reasonable assumption given the tiny costs associated with trading derivatives in liquid markets. The distribution of gambling outcomes can be any that satisfies these constraints, since derivatives are much more flexible instruments for gambling compared to asset substitution.

We also have in mind that the gambling is very short-term, which is essential if the proceeds are to be used to pay current liabilities. Very short-term gambles are definitely possible when gambling uses derivatives. Very short-term gambling means it is reasonable that the liquidation value after the gamble is assumed to be known before the gamble. As noted above, short-term gambling motivates making L an exogenous constant which follows from the firm's gambling counterparties understanding the firm's marketability in the short run.

The constraint of gambling outcomes in (3) defines the maximum amount the firm can

[17] See Jagannathan (1984).

lose in the gamble. If gambling is not available, the outcome is always 0. If the gambling is possible, the gambling counterparties would only gamble with the firm if the firm can credibly repay, that is to say, there is a limit that the firm can promise to lose in a gamble. This limit, π plus the fraction γ of the liquidation value, is determined by the the priority of the gambling counterparties. Absent superpriority, the firm can always promise the cash flow π to the gambling counterparties by paying upfront, but the other assets go to the bondholders, so that $\gamma = 0$. With superpriority, gambling counterparties are paid before the existing debt F, and would receive a fraction γ of assets not serving as perfected collateral. Hence, superpriority increases γ, allowing larger gambles. In the stripped-down model, we assume that superpriority increases the amount available for gambling from 0 to the entire liquidation value L, so $\gamma = 1$ with superppriority and $\gamma = 0$ without. If some but not all assets are protected against superpriority claims, we could have $0 < \gamma < 1$. The multi-period model in Section 3 allows γ to take on any value in $[0,1]$.

In (3), we assume an upper bound $\bar{G} >> 0$ to avoid a closure problem in some cases if there is no upper limit of gambling, but we can compute limits of expected payoffs as $\bar{G} \uparrow \infty$. We think of $\bar{G} >> 0$, but we require at a minimum that $\bar{G} \geq F - B - \pi$, or $\bar{G} + \pi \geq F - B$, meaning that gambling to cover the required payment is possible.

Then, given the owners' choice of gambling $\mathbf{G}(\tilde{x})$, the bond value is

$$\text{bond value} = \mathrm{E}\Big[(\pi + B + \mathbf{G}(\tilde{x}) < F)(1-c)(\pi + \mathbf{G}(\tilde{x}) + L) + (\pi + B + \mathbf{G}(\tilde{x}) \geq F)F\Big].$$

Note that, $\pi + \mathbf{G}(\tilde{x}) + L \geq \pi - \gamma L - \pi + L \geq (1-\gamma)L \geq 0$. Since $L \leq B$ by assumption 1, we also have that $\pi + \mathbf{G}(\tilde{x}) + L \leq F$ when $\pi + \mathbf{G}(\tilde{x}) < F - B$. Therefore, the bondholders are better off if the firm survives, which is appropriate for asking whether gambling for redemption is better for bondholders than not gambling. Below, we show that the gambling behavior of the owners is quite different depending on whether F is greater than $C - \gamma L$. In particular, if $F < C - \gamma L$, the owners will gamble at a small scale which benefits bondholders; whereas if $F > C - \gamma L$, the owners will gamble at a large scale which rips off the bondholders. With superpriority, γ is larger, implying that the owners gambles for ripoff more often.

Example 1: gambling for redemption (absent superpriority, $\gamma = 0$)

Without superpriority, $\gamma = 0$. Figure 1 demonstrates the equity value and bond value as functions of cash flow when $F < C$, i.e., the face value of debt is less than the firm's continuation value. The blue lines represent the values without gambling: if cash flow π is below $F - B$, the firm loses all the continuation value in bankruptcy and bondholders lose a fraction $1 - c$ of the remaining assets $\pi + L$; if cash flow π is above $F - B$, the net continuation value $C - B$ is maintained and the total bond value is F.

To reach the maximal expected equity value, the value of an optimal gambling strategy, shown by the red lines, should "concavify" the blue curves. As shown in Figure 1, if the firm starts with cash flow $\pi_2 \geq F - B$, the firm is sound and the owners will only gamble along the 45 degree segment and will never gamble down below $F - B$, and none of those gambles change the expected payoff for the owners or bondholders. However, if the firm starts with cash flow $\pi_1 < F - B$, equity is worthless unless there is gambling. In this case, optimal gambling can be written as

$$\mathbf{G}(\tilde{x}) = \begin{cases} \xrightarrow{\frac{\pi_1}{F-B}} F - B - \pi_1 \\ \xrightarrow{\frac{F-B-\pi_1}{F-B}} -\pi_1, \end{cases}$$

where the gambling outcomes are chosen to pay off the debt in the up state and consume the cash in the down state, and the probabilities are chosen to make $\mathbf{E}[\mathbf{G}(\tilde{x})] = 0$. These choices give the optimal gamble that retains the continuation value as often as possible. This gambling always has two outcomes, which implies that it can be implemented using a digital option: if a commodity has continuous cdf $F(y)$, we can pick y^* such that $F(y^*) = \frac{F-B-\pi_1}{F-B}$, then use π_1 to buy a digital option that pays $F - B$ when $y \geq y^*$ and 0 when $y < y^*$.

To conclude, gambling for redemption adds value to the owners and the bondholders due

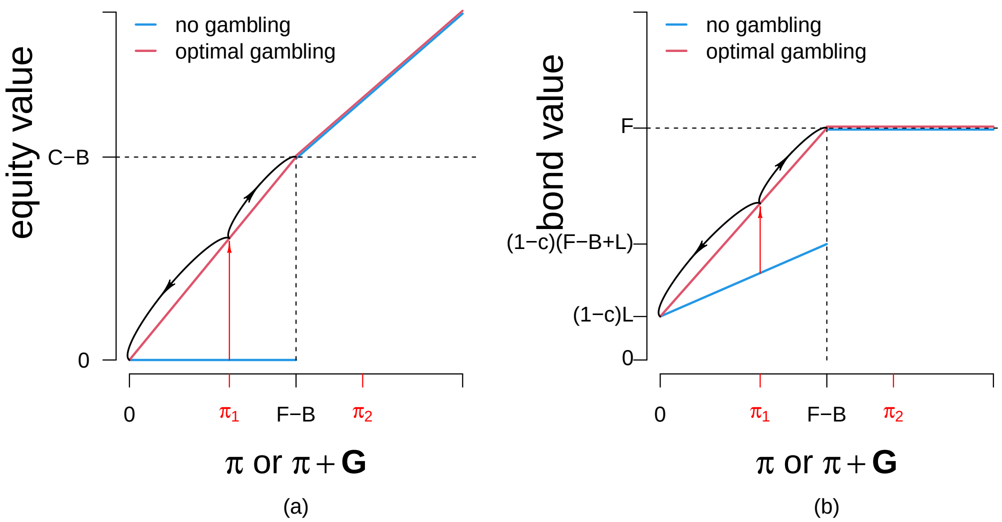

Figure 1: Gambling for redemption, which is just enough to pay off debt, makes both equity and bonds more valuable. The owners optimally gamble for redemption when the required debt payment is less than equity's share of the firm's continuation value. To stay alive, cash (= cash flow $\pi +$ any gambling proceeds $\mathbf{G}$) must cover the required debt payment (maturing debt F less value of new debt B). If the firm stays alive, the equity value is the continuation value C plus any leftover cash $\pi + G - F$, while the existing bondholders get F. If the firm fails, the bondholders receive the liquidation value of the assets L plus any cash $\pi + g$, all subject to a proportional bankruptcy cost c. Expected equity value after optimal gambling is given by the red lines, which"concavifies" the equity value before gambling (blue lines). When cash flow is π_1 (which does not cover the required debt payment), the optimal gambling shown by the curved black arrows is gambling from π_1 to $F - B$ with probability $\frac{\pi_1}{F-B}$ and from π_1 to 0 with probability $1 - \frac{\pi_1}{F-B}$. Expected equity and bond values both increase as indicated by the red arrows. For $\pi_2 \geq F - B$ (the firm has enough cash flow to cover required debt payment), optimal gambling does not change expected equity and bond values.

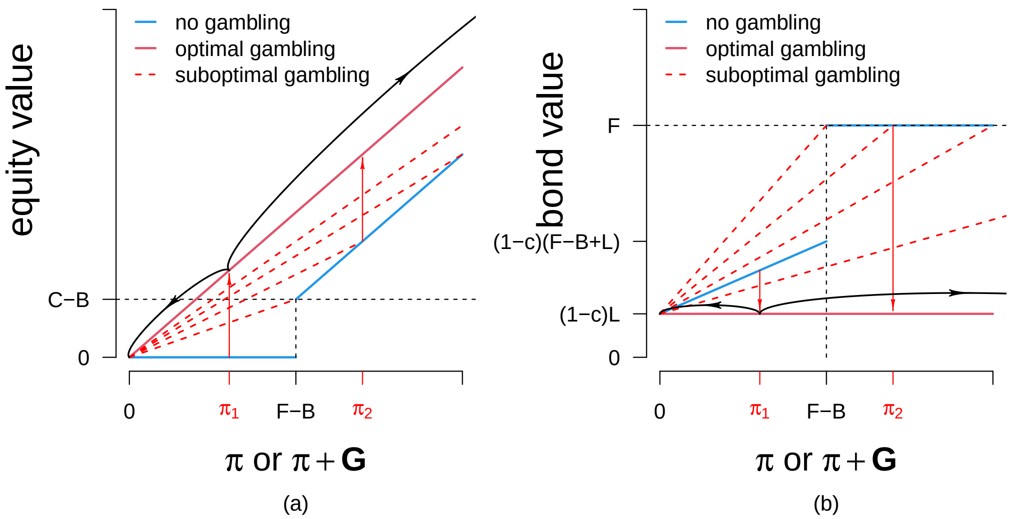

Figure 2: Gambling for ripoff, which is gambling to a large scale to fail as often as possible, benefits the owners at the expense of bondholders. The owners optimally gamble for ripoff when the required debt payment is greater than the equity's share of the firm's continuation value. Similar to gambling for redemption, expected equity value after optimal gambling (solid red line) is a concavification of the equity value before gambling (blue lines). Interestingly, whether the firm is out of money ($\pi = \pi_1$) or in the money ($\pi = \pi_2$), the owners always choose to gamble for ripoff. For any positive cash flow π, the optimal gambling shown by the black arrows is gambling from π to $\bar{G} + \pi$ with probability $1 - \frac{\pi}{\bar{G}+\pi}$ and from π to 0 with probability $1 - \frac{\pi}{\bar{G}+\pi}$. As $\bar{G}$ goes to infinity, the bond value decreases, as indicated by the red arrow, to $(1-c)L$, the amount L that cannot be gambled away by the owners net of bankruptcy cost cL.

to less frequent value loss in bankruptcy. When there is no fractional bankruptcy cost ($c = 0$) and the face value of ongoing debt is equal to the liquidation value ($B = L$), the bondholders are indifferent to whether the owners gamble or not.

Example 2: gambling for ripoff (absent superpriority, $\gamma = 0$)

However, when the face value of debt F is greater than the firm's continuation value C, gambling for redemption is no longer optimal. The dashed red lines in Figure 2 give the payoffs of fair Bernoulli gambles. An example is

$$\mathbf{G}(\tilde{x}) = \begin{array}{l} \xrightarrow{\pi/(\bar{G}+\pi)} \ \bar{G} \\ \xrightarrow[\bar{G}/(\bar{G}+\pi)]{} \ -\pi, \end{array}$$

where the gambling outcomes are chosen to be as large as possible in the up state and consume all the cash in the down state, and the probabilities are chosen such that $\mathrm{E}[\mathbf{G}(\tilde{x}] = 0$. These choices give the optimal gamble that loses the continuation value as often as possible, in contrary to gambling for redemption. Similarly, this gamble can also be implemented using a digital option as described in Example 1: if a commodity has continuous cdf $F(y)$, we can pick y^* such that $F(y^*) = \frac{\bar{G}}{\bar{G}+\pi}$, then use π to buy a digital option that pays $\bar{G}+\pi$ when $y \geq y^*$ and 0 when $y < y^*$.

As the payoff $\bar{G}$ increases, the probability of winning declines but the owners have a larger value because not paying $F-B$ is more important to them than not receiving $C-B$. The above Bernoulli gamble concavifies the owners' original value function and is the optimal gamble. In this gamble, the owners obtain $\bar{G}$ with probability $\frac{\pi}{\bar{G}+\pi}$ and 0 with probability $1-\frac{\pi}{\bar{G}+\pi}$. Therefore, the maximum that the owners can achieve in gambling in the limit is $\lim_{\bar{G}\uparrow\infty} \frac{\pi}{\bar{G}+\pi}(\bar{G}-F+C) = \pi$ and the bond value is $\lim_{\bar{G}\uparrow\infty} \left(\frac{\pi}{\bar{G}+\pi}F + \frac{\bar{G}}{\bar{G}+\pi}(1-c)L\right) = (1-c)L$. The bondholders almost always receive the liquidation value less bankruptcy cost, and never receive more. In this case, gambling for redemption would increase the total value of bond and equity because the continuation value would be preserved as often as possible, but the owners would rather choose a larger gamble which gives them a higher value at the expense of bondholders. We have shown that for $\pi = \pi_1$, if the firm has cash flow $< F-B$, gambling for ripoff transfers value from the bondholders to the owners. Interestingly, gambling for ripoff is also optimal for equity in this example even if $\pi = \pi_2 > F-B$, a case in which the firm has enough to pay off the debt $F-B$ without gambling.

Example 3: with superpriority $C - L < F < C$

Positive available liquidation value to gamble will change the shape of gambling if $C - L < F < C$, as illustrated in Figure 3. Superpriority makes the liquidation value available for gambling, allowing firm owners to gamble down to $-L$ instead of 0. In Figure 3, the continuation value

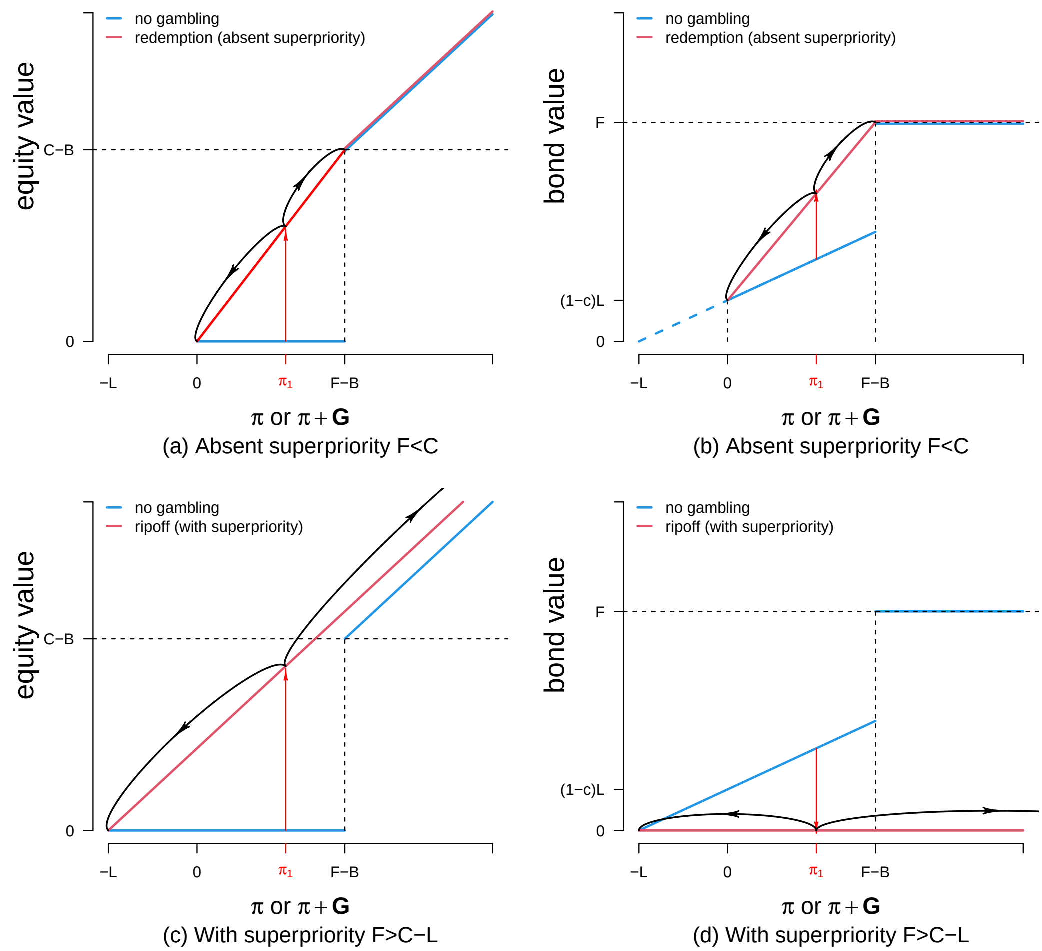

Figure 3: When $C - L < F < C$, superpriority makes gambling for ripoff more appealing to the owners. Absent superpriority, the owners can only gamble down to 0 and will gamble for redemption since $F < C$, shown by (a)(b); with superpriority, the owners can gamble down to $-L$ and will choose to gamble for ripoff since $L + F > C$, shown by (c)(d).

C is greater than the face value of debt F, so that the owners will gamble its cash flow for redemption of the value of the payment due absent superpriority and obtain $\frac{\pi}{F-B}(C + \pi - F)$, as depicted by Figure 3(a). Figure 3(b) shows the relevant bond value and bondholders are also better off. However, when gambling further down to $-L$ is available, gambling for ripoff yields greater benefits as shown in Figure 3(c). However, this larger gambles make bondholders worse as in Figure 3(d). In the graph, whether $L+F-B$ is greater than $C-B$ (or F is greater than $C-L$) determines the optimal gambling, where $C-L$ is the owners' bankruptcy cost, but they benefit from bankruptcy by not paying F.

These graphic observations are formally stated by the following propositions:

PROPOSITION 2.1 when $F < C-\gamma L$ (the payment due now is less than the value lost in bankruptcy), it is optimal to gamble for redemption. Under this parameter restriction, gambling increases the value of both bond and equity when $\pi < F-B$, and leaves both unchanged when $\pi \geq F-B$. Specifically,

(1) If the cash flow before gambling is insufficient to make the current debt payment ($\pi < F-B$), all optimal gambles have the same distribution. In particular, an optimal gamble is

$$\mathbf{G}^*(\tilde{x}) = \begin{cases} F-B-\pi, & \text{for } 0 < x \leq \frac{\pi+\gamma L}{F-B+\gamma L} \\ -\gamma L-\pi, & \text{for } \frac{\pi+\gamma L}{F-B+\gamma L} < x < 1. \end{cases} \tag{4}$$

(2) If the cash flow before gambling is sufficient to make the current debt payment ($\pi \geq F-B$), the optimal gambles are the feasible gambles that never reduce cash below the current debt payment $F-B$. The set of solutions is

$$\left\{\mathbf{G}:[0,1]\rightarrow\mathscr{O}\;\middle|\;\pi+\mathbf{G}(\tilde{x})\geq F-B \text{ and } \mathrm{E}[\mathbf{G}(\tilde{x})]=0\right\}.$$

In particular, not gambling ($\mathbf{G}^*(\tilde{x})\equiv 0$) is always optimal, and it is the only solution if $\pi = F-B$.

(3) The expected payoffs are

$$\text{equity value} = \begin{cases} \pi - F + C, & \text{for } \pi \geq F - B \\ \frac{\pi+\gamma L}{F-B+\gamma L}(C-F), & \text{for } \pi < F - B \end{cases}$$

$$\text{bond value} = \begin{cases} F, & \text{for } \pi \geq F - B \\ \frac{\pi+\gamma L}{F-B+\gamma L}F + (1 - \frac{\pi+\gamma L}{F-B+\gamma L})(1-c)(1-\gamma)L, & \text{for } \pi < F - B \end{cases}$$

$$\text{bond+equity} = \begin{cases} \pi + C, & \text{for } \pi \geq F - B \\ \frac{\pi+\gamma L}{F-B+\gamma L}C + (1 - \frac{\pi+\gamma L}{F-B+\gamma L})(1-c)(1-\gamma)L, & \text{for } \pi < F - B \end{cases}$$

Proof. (Sketch) Following Aumann and Perles (1965), we first concavify the objective function and use Kuhn-Tucker conditions to solve the concavified problem. Since the constructed solution for the concavified problem is also feasible for the original problem, and the concavified objective function is greater than the original function, we can conclude that the solution(s) for the concavified problem also solves the original problem. For details, see Appendix A. ■

In Proposition 2.1(2), if we believe that gambling is costly, or if we are not using risk neutral probabilities (the owners are risk averse), then $\mathbf{G}^*(\tilde{x}) \equiv \pi$ (no gambling) should be the unique solution. However, in Proposition 2.1(1), gambling is still optimal in the face of a sufficiently small cost.

PROPOSITION 2.2 when $F > C - \gamma L$, it is optimal for the owners to gamble for ripoff. Gambling for ripoff transfers value from bondholders to the owners when $\pi < F - B$, and also destroys continuation value when $\pi \geq F - B$. Specifically,

(1) The optimal gambling is

$$\mathbf{G}^*(\tilde{x}) = \begin{cases} \bar{G}, & \text{for } 0 < x \leq \frac{\pi+\gamma L}{\bar{G}+\pi+\gamma L} \\ -\gamma L - \pi, & \text{for } \frac{\pi+\gamma L}{\bar{G}+\pi+\gamma L} < x < 1 \end{cases} \tag{5}$$

(2) Owners' payoff is $\frac{\pi+\gamma L}{\bar{G}+\pi+\gamma L}(C+\pi+\bar{G}-F)$, which increases to $\pi+\gamma L$ as $\bar{G}\to\infty$. The value of the bond is $\frac{\pi+\gamma L}{\bar{G}+\pi+\gamma L}F + \frac{\bar{G}}{\bar{G}+\pi+\gamma L}(1-c)(1-\gamma)L$, and declines to $(1-c)(1-\gamma)L$ as $\bar{G}\to\infty$. For any $\pi > 0$, the total value of bond and equity is always $\pi+L-(1-\gamma)cL$ when $\bar{G}\to\infty$.

Proof. See Appendix B. ■

Since it is efficient to continue the firm, gambling for redemption maximizes the probability of continuation and is socially beneficial, while gambling for ripoff minimizes this probability and is socially damaging. In the trade-offs between gambling for redemption and gambling for ripoff, superpriority plays an important role. It transfers owners the liquidation value which should go to bondholders, making gambling for ripoff more appealing to the owners. With more "ripoff" cases, continuation value is lost more often.

There is also a knife-edge case when $F = C - \gamma L$. In this case, any fair gamble with outcomes distributed long the 45-degree linear segment would yield the same expected value. That is to say, gambling for redemption and gambling for ripoff give the same outcome for the owners, and anything in between the two polar cases is also optimal. Though these optimal gambles generate different values for the bondholders (for example, we still have gambling for redemption makes the bondholders better-off and gambling for redemption worse-off), we don't want to go into the details of what the equilibrium (equilibria) is (are) because it is reasonable to believe that $F = C - \gamma L$ almost never happens.

In this paper we assume the absence of workouts before or during bankruptcy. We can think of workouts as infeasible if there is a large number of diverse claimants. Even if workouts are possible, gambling for ripoff tends to be robust because the owners would prefer gambling to workouts and bankruptcy. Under gambling for ripoff, the bondholders receive little assets especially when superpriority is available. In a hypothetical workout, this threat of gambling for ripoff allows the owners to appropriate most of the bondholders' value, making the payoffs similar to gambling for ripoff even if the gambling does not actually happen in equilibrium. Gambling for redemption is less robust to the availability of workouts, but in reality the advantage of workouts can be undermined by the high costs.

2.1 Applications

The single-period model, despite its simplicity, can be a useful tool for understanding gambling behaviors under various economic and legislative conditions. We present four cases where this framework may be useful to elucidate the observed data.

Traczynski (2019) documents empirical evidence that in states where antidiscrimination laws permit married firm owners to select asset protection at times of failure, firms receive smaller loans without taking on additional risks. This finding aligns with our model. While asset protection increases the benefit of failure, it restricts the amount that the firm can risk in gambling, which is contrary to superpriority laws. As a result, the tradeoff between gambling for redemption or ripoff remains unchanged. But because owners accumulate more assets upon failure, bondholders would receive less in either case, resulting in reduced borrowing.

The introduction of superpriority for retiree medical benefits may also increase the incentives for gambling. After Congress enacted Chapter 11 section 1114 in 1988, granting special priority to retiree medical benefits, gambling for ripoff could become more attractive because an increase in debt obligations would reduce the net gain that the owners receive in continuation. Consequently, gambling for ripoff would wipe out most of the firm's assets, and the dilution effect of the assets could be more severe than merely taking on additional debt. Such high-risk behavior might make it more difficult for the firm to obtain funding, and bondholders may only be willing to lend if the firm promises to file for Chapter 7 liquidation to circumvent the legislation with underfunded retiree insurance benefits, exacerbating the problems highlighted by Keating (1990, 1991).

Dambra et al. (2023) provides empirical evidence that multi-employer pension plans in the United States have exhibited increased risk-taking behavior since the enactment of the American Rescue Plan Act in 2021. This legislation infused funds into specific underfunded multi-employer pension plans, and its impact aligns with our model prediction. Specifically, for underfunded pension plans that face difficulties in meeting their funding obligations, a bailout could amplify the perceived value of failure, encouraging plan management to engage in riskier investments.

3 The Dynamic Model with Endogenous Debt and Continuation Value

Our analysis so far has been based on a single-period model, which is meant to capture the conditions for gambling for redemption and ripoff. While the assumptions are intended to look like a snapshot of a dynamic model, the exogenous amount of borrowing in the model may not be an optimal choice. Instead, the single-period model solves for the owners' risk-taking problem when superpriority laws are a surprise, and the amount of borrowing comes from the regime before superpriority. If the superpriority legislation induces gambling for ripoff in the single-period model, it is likely to do so in a multi-period model as well. Given that, the continuation value is likely to be small, and this will affect how much the firm can borrow and the terms on which it can borrow.

We now shift our focus to a dynamic model in which continuation value and the amount of borrowing are endogenous. Both are affected by the availability of gambling in general and the presence or absence of superpriority law in particular. Lenders are able to anticipate the owners' behavior in response to superpriority, and we find that a significant availability of superpriority tends to reduce firm value. This is because the increased incentive to gamble is anticipated and reflected in the terms on which the owners can borrow.

In this section's model, the firm is liquidated and ceases to exist either due to bankruptcy resulting from failure to meet debt payments, or due to an exogenous disappearance of the firm's market. All debt has a one-period duration and is priced at a fair value considering any dilution. To avoid bankruptcy, owners are required to repay fully the face value of the debt.[18] The exogenous disappearance of the firm's market occurs with a conditional probability ρ, after gambling and before borrowing, with disappearance drawn independently of the other shocks in the model. We write the objective function as an expectation taken over gambling and the distribution of exogenous ending dates (due to disappearance of the firm's market) and endogenous ending dates (due to bankruptcy). We consider the value function at two types of choice nodes: choice of gambling (before gambling) and proposal of new debt (after gambling). The choice variables, state variables, and realization of shocks are conditional on the firm still existing.

Here is the timeline of the model at time t:

t	*I. capital K_t and face value of debt F_t come from time $t-1$*
	II. exit with exogenous probability ρ
	III. cash flow vK_t arrives
$C^{b4}(K_t, F_t)$ ←	*IV. owners choose gambling $\mathbf{G}(\cdot)$*
	V. gambling outcome $\mathbf{G}(x_t)$ implies "paper surplus" $S_t \equiv vK_t - F_t + \mathbf{G}(x_t)$
$C^{after}(K_t, S_t)$ ←	*VI. owners choose borrowing (B_t, F_{t+1}) acceptable to investors with $K_t + S_t + B_t \leq (1+\bar{\imath})K_t$*
	VII. if $S_t + B_t < 0$, not enough cash to cover F_t, terminate
	VIII. purchase of new capital and shock δ_t, capital after the shock $K_{t+1} \equiv \delta_t(K_t + S_t + B_t)$
$t+1$	*IX. carry capital K_{t+1} and debt F_{t+1} into the next period*

In period t, the firm begins with capital K_t and maturing debt F_t, where $-F_t$ is cash if

[18] This approach finesses the leverage ratchet effect by Admati et al. (2018) and DeMarzo and He (2021), which suggest that existing bondholders are the only ones who benefit from a debt buyback.

Figure 4: Timeline

$F_t < 0$. With exogenous constant probability ρ, nature terminates the firm and the owners receive $(K_t - F_t)^+$, while bondholders receive $\min\{K_t, F_t\}$. With probability $1-\rho$, the firm is not terminated, capital pays a cash flow $vK_t > 0$ to the owners, where v is a constant.

After continuation, the owners have the option to participate in a frictionless competitive gambling market. The value function $C^{b4}(K_t, F_t)$ is the owners' continuation value "before" gambling, as indicated by the arrow before step IV in Figure 4. In a frictionless competitive gambling market, the gambling choice $\mathbf{G}(\tilde{x})$ has a mean of 0, representing a fair gamble as in the single-period model, and $\mathbf{G}(\tilde{x}) \equiv 0$ if there is no gambling.

Without superpriority, the owners can gamble with only the cash flow $vK_t + (F_t)^-$,[19] which includes savings (negative borrowings) from the previous period. With superpriority, the owners can gamble with $vK_t + (F_t)^- + \gamma\theta K_t$, where the positive parameter γ is the proportion of assets not protected, such as those not pledged as perfected collateral.[20] The liquidation value per unit of capital is denoted by $\theta \in [0,1)$. So, $\gamma\theta K_t$ is the value of capital that the owners can gamble away.

The proceeds from new borrowing cannot be used to pay off gambling debts. Similar to the single-period model, $\bar{G}K_t$ is the upper bound for gambling to prevent a closure problem. Note that $\gamma = 0$ if there is no superpriority. Formally, the set $\mathscr{G}_t$ of feasible gambles is given by

$$\mathscr{G}_t \equiv \left\{ \text{non-increasing } \mathbf{G} : [0,1] \to \mathscr{O}_t \,\middle|\, \mathrm{E}[\mathbf{G}(\tilde{x})] = 0 \right\}, \tag{6}$$

[19]We follow the convention that for any $a \in \mathbb{R}$, $a^+ \equiv \max\{a, 0\}$ and $a^- \equiv (-a)^+$. Note that $a = a^+ - a^-$, $a^+, a^- \geq 0$, and $a^+ a^- = 0$.

[20]For our analysis γ is exogenous, but in a richer model superpriority laws could induce firms to undertake otherwise inefficient actions to increase γ since the owners and bondholders would have incentives to seek protection of the firm's assets.

given the feasible gambling outcomes

$$\mathscr{O}_t = \begin{cases} \{0\}, & \text{no gambling} \\ [-\nu K_t - (F_t)^- - \gamma\theta K_t, \bar{G}K_t], & \text{otherwise} \end{cases} \tag{7}$$

It is worth emphasizing that gambling in this paper has a short duration. We think that it is optimal for the owners to use short-maturity derivatives, as they would otherwise need to manage the risk of the various positions over the course of the gamble's duration.

After the gambling outcome is realized, the owners have "paper surplus" $S_t \equiv \nu K_t - F_t + \mathbf{G}(x_t)$, which assumes the debt will be paid in full. The value function $C^{after}(K_t, S_t)$ represents the owners' continuation value "after" gambling, denoted by an arrow after step IV in Figure 4.

Next, the owners propose a bond offer with borrowing B_t and face value F_{t+1}. If $B_t < 0$, it is interpreted as risk-free investment. The bond market is frictionless and competitive, which implies that the owners can without loss of generality propose an offer that will be accepted by the investors. Any offer that is not accepted can be replaced by $B_t = 0$ and $F_{t+1} = 0$.

If there is not enough cash after borrowing to cover all the debt due, i.e. $S_t + B_t < 0$, bankruptcy occurs. In this case, the new debt issuance is cancelled, and the owners receive $(1-c)(\theta K_t + S_t)^+$ which is the value of assets after selling at a discounted price and a deduction of fractional bankruptcy cost. Existing bondholders obtain $(1-c)[F_t \wedge (\mathbf{G}(x_t) + \theta K_t + \nu K_t)]$ if borrowing is positive.[21]

If $S_t + B_t \geq 0$, there is enough cash to clear all debt obligations, the firm continues and may also increase the capital at a growth rate capped by $\bar{i}$ per period. We impose the cap $\bar{i}$ to rule out infinite borrowing and to reflect the fact that firms typically have limited capacity

[21] If F_t is negative, the firm lends the money and will recover $-F_t$ in full. If F_t is positive, we assumes that bondholders are also subject to a fractional bankruptcy cost even after they received full repayment. Since $\mathbf{G}(\tilde{x}) \geq -\nu K_t - (F_t)^- - \gamma\theta K_t$, the bondholders at least receive $(1-c)[F_t \wedge (1-\gamma)\theta K_t]$ where $(1-\gamma)\theta K_t$ can be seen as the protected collateral not eligible for gambling.

to expand within a period of time. The new capital after augmentation but before the shock $\tilde{\delta}$ is $K_t + S_t + B_t$, which is the sum of remaining capital and new investment that comes from net cash and new borrowing. The firm must ensure that it can repay the face value of the new debt after borrowing, while respecting the maximum growth rate $\bar{i}$ of capital so that

$$K_t \leq K_t + S_t + B_t \leq (1 + \bar{i})K_t \tag{8}$$

At the end of the period, capital is subject to an i.i.d. multiplicative shock $\tilde{\delta}_t > 0$, so capital after the shock is given by

$$K_{t+1} = \delta_t (K_t + S_t + B_t), \tag{9}$$

which is the capital the owners carry on into the next period. The shock $\tilde{\delta}_t$ captures any depreciation and other exogenous factors that affect the firm's asset value, such as changes in the economy, technology, or market conditions, and it is assumed to be independent and identically distributed over time. The owners then face the same decision problems as before, starting with the new level of capital and new debt obligations.

In addition, we make two assumptions about the parameters:

Assumption 2 $(1 + \bar{i})(1 - \rho)\mathrm{E}[\tilde{\delta}_t] < 1$.

Assumption 3 $(1 + \nu(1 - \rho))\mathrm{E}[\tilde{\delta}_t] > 1$.

Assumption 2 ensures that firm's value is finite. Assumption 3 shows a preference for new capital investment over cash retention, given that $(1 + \nu(1 - \rho))\mathrm{E}[\tilde{\delta}_t]$, i.e. capital has a large average return on investment than cash.

There are several points in a period where we could examine the equity's continuation values, but it is simplest to focus on nodes right before and after gambling. We will present the owners' problems sequentially in the form of a Bellman equation. We look for an equilibrium that is Markov in a short list of state variables. Specifically, the owners' value function C^{b4} before gambling depends on the firm's capital after a shock, K_t, and its outstanding debt,

F_t. The value function C^{after} after gambling depends on the capital after the shock, K_t, and the net cash after gambling realization, S_t. We will also restrict attention to equilibria that are scale-independent.

3.1 Bellman equations

We analyze a subgame perfect Nash equilibrium in which all the optimal debt offers are accepted. We state the owners' problems before gambling (gambling node) and after gambling but before borrowing (borrowing proposal node):

(Gambling node) At time t after surviving the exogenous termination shock, given capital K_t and debt outstanding F_t, the owners choose adapted gambling $\mathbf{G}(\tilde{x}) \in \mathscr{G}$ to maximize expected value. The Bellman equation is

$$C^{b4}(K_t, F_t) = \max_{\mathbf{G}\in\mathscr{G}_t} \mathrm{E}\left[C^{after}(K_t, \nu K_t - F_t + \mathbf{G}(\tilde{x}))\right], \tag{10}$$

subject to the set of feasible gambles $\mathscr{G}_t$ defined in (6) and (7).

(Borrowing proposal node) At time t after gambling is realized, given capital after gambling K_t and net cash after gambling $S_t \equiv \nu K_t - F_t + \mathbf{G}(x_t)$, the owners choose adapted new borrowing and face value (B_t, F_{t+1}) to maximize expected value. The Bellman equation is

$$\begin{aligned} C^{after}(K_t, S_t) = \max_{(B_t, F_{t+1})} \mathrm{E}\Big[& \big(S_t + B_t < 0\big)(1-c)(\theta K_t + S_t)^+ + \big(S_t + B_t \geq 0\big)\Big\{\rho(K_{t+1} - F_{t+1})^+ \\ & + (1-\rho)C^{b4}(K_{t+1}, F_{t+1})\Big\}\Big], \end{aligned} \tag{11}$$

subject to the borrowing constraint (8), capital augmentation (9), and the bondholders' participation constraint, which is either (lending case) $B_t \leq 0$ and $B_t \leq F_{t+1}$, or (borrowing case) $B_t > 0$ and

$$B_t \leq \mathrm{E}\left[\rho(F_{t+1} \wedge K_{t+1}) + (1-\rho)\left\{\left(1-\mathscr{I}^*_{t+1}\right)(1-c)[F_{t+1} \wedge (\mathbf{G}^*(\tilde{x}, K_{t+1}, F_{t+1}) + \theta K_{t+1} + vK_{t+1})] + \mathscr{I}^*_{t+1} F_{t+1}\right\}\right].$$

In this equation, the survival indicator $\mathscr{I}^*_{t+1}$ and the surplus $\tilde{S^*}_{t+1}$ are derived from the owners' equilibrium policies $B^*_{t+1}(K_{t+1}, \tilde{S}_{t+1})$ and $\mathbf{G}^*(\tilde{x}, K_{t+1}, F_{t+1})$ in the next period. Specifically,

$$\mathscr{I}^*_{t+1} = \begin{cases} 1, \text{ if } \tilde{S}_{t+1} + B^*_{t+1}(K_{t+1}, \tilde{S}_{t+1}) \geq 0 \\ 0, \text{ otherwise} \end{cases}$$

and

$$\tilde{S^*}_{t+1} \equiv vK_{t+1} - F_{t+1} + \mathbf{G}^*(\tilde{x}, K_{t+1}, F_{t+1}).$$

We assume that the bond market does not deal with a firm that is facing bankruptcy in the current period. That is, when the proposed new borrowing does not cover the shortfall, we let $\iota = 0$. We also assume that the firm does not have other sources of financing.

To simplify the analysis, we normalize the problems by dividing K_t throughout, assuming homogeneity of the problems. This allows us to expressed everything as value per unit of capital.

3.2 Normalization

Given that we are looking for a scale-independent solution in our short list of state variables in each node, it is useful to define the following normalized variables:

$$\delta \equiv \delta_t,\ \delta' \equiv \delta_{t+1};\ s \equiv \frac{S_t}{K_t},\ s' \equiv \frac{S_{t+1}}{K_{t+1}};$$
$$\beta \equiv \frac{B_t}{K_t},\ \phi \equiv \frac{F_t}{K_t},\ \phi' \equiv \frac{F_{t+1}}{K_{t+1}};\ b \equiv \frac{B_t}{K_{t+1}/\delta_t},\ f' \equiv \frac{F_{t+1}}{K_{t+1}/\delta_t} = \delta\phi',$$

and the ratio of capital augmentation derived from (9) is

$$\frac{K_{t+1}/\delta_t}{K_t} = 1 + s + \beta \in [1, 1 + \bar{i}] \tag{12}$$

To clarify the notation, we use a prime symbol ($'$) to denote the variables in the next period, and the variables without a prime represents the current period. For example, s' is net cash per capital after gambling in the next period and s in this period. The choice variables are denoted as (β, f'), representing the borrowing per new capital and the face value per new capital, respectively.

By the assumption of homotheticity, we have

$$\begin{aligned}
&C^{b4}(K_t, F_t) = K_t C^{b4}(1, \frac{F_t}{K_t}) = K_t C^{b4}(1, \phi) \equiv K_t C^{b4}(\phi), \\
&C^{after}(K_t, S_t) = K_t C^{after}(1, \frac{S_t}{K_t}) = K_t C^{after}(1, s) \equiv K_t C^{after}(s), \\
&\mathbf{G}(\tilde{x}, K_t, F_t) = K_t \mathbf{G}(\tilde{x}, 1, \frac{F_t}{K_t}) = K_t \mathbf{G}(\tilde{x}, 1, \phi) \equiv K_t \mathbf{g}(\tilde{x}, \phi)
\end{aligned}$$

Now we restate the firm's problem:

(Gambling node) Given debt per unit of capital ϕ, the owners choose adapted gambling $\mathbf{g}(\tilde{x}) \in \mathscr{G}_\phi$ to maximize expected value. Restating the Bellman equation (10) in terms of the normalized variables, we have

$$C^{b4}(\phi) = \max_{\mathbf{g} \in \mathscr{G}_\phi} \mathrm{E}\left[C^{after}(v - \phi + \mathbf{g}(\tilde{x})) \right], \tag{13}$$

the set $\mathscr{G}_\phi$ of feasible gambles is given by

$$\mathscr{G}_\phi \equiv \left\{ \text{non-increasing } \mathbf{g} : [0,1] \to \mathscr{O}_\phi \,\middle|\, \mathrm{E}[\mathbf{g}(\tilde{x})] = 0 \right\},$$

given the feasible gambling outcomes

$$\mathscr{O}_\phi = \begin{cases} \{0\}, & \text{no gambling} \\ [-v - \phi^- - \gamma\theta, \bar{G}], & \text{otherwise} \end{cases}$$

Note that $\gamma = 0$ if there is no superpriority.

(Borrowing proposal node) Given net cash per capital after gambling s, the owners choose adapted new borrowing and face value (β, f') to maximize expected value. Restate the Bellman equation (11) in terms of normalized variables, we have

$$C^{after}(s) = \max_{(\beta,f')} \big(s+\beta<0\big)(1-c)(\theta+s)^+ + \big(s+\beta\geq 0\big)(1+s+\beta)\mathrm{E}\Big[\rho\tilde{\delta}(1-\tilde{\phi}')^+ + (1-\rho)\tilde{\delta}C^{b4}(\tilde{\phi}')\Big], \tag{14}$$

subject to the borrowing constraint derived from (8):

$$-s \leq \beta \leq \bar{i} - s \tag{15}$$

and the bondholders' participation constraint, which is either (lending case) $\beta \leq 0$ and $b \leq f'$, or (borrowing case) $\beta > 0$ and

$$b \leq \mathrm{E}\Big[\rho\tilde{\delta}(\tilde{\phi}' \wedge 1) + (1-\rho)\tilde{\delta}\Big[(1-\mathscr{I}'^*)(1-c)\big[\tilde{\phi}' \wedge \big(\mathbf{g}^*(\tilde{x},\tilde{\phi}') + \theta + v\big)\big] + \mathscr{I}'^*\tilde{\phi}'\Big]\Big], \tag{16}$$

where

$$\tilde{\phi}' = \frac{f'}{\tilde{\delta}},\ b = \frac{\beta}{1+s+\beta}.$$

In this equation, the survival indicator $\mathscr{I}'^*$ and the surplus $\tilde{s'}^*$ are derived from the owners' equilibrium policy $\boldsymbol{\beta}'^*$ and $\mathbf{g}^*(\tilde{x}, \tilde{\phi}')$ in the next period. Specifically

$$\mathscr{I}'^* = \begin{cases} 1, \text{ if } \tilde{s}'^* + \beta'^* \geq 0 \\ 0, \text{ otherwise} \end{cases}$$

$$\tilde{s}'^* = v - \tilde{\phi}' + \mathbf{g}^*(\tilde{x}, \phi').$$

4 Multi-period Equilibrium and Graphic Illustration

Now we turn to an economic analysis of the multi-period model, using a numerical solution.[22] The assumed parameter values, given in Table 2, are chosen to obtain a solution with risky debt, which is the interesting case. Specifically, we assume a wide range of $\tilde{\delta}$'s, with a uniform

$\bar{i}$	0.03
ρ	0.15
c	0.05
$\tilde{\delta}$	$U(0.05, 1.95)$
v	0.05
θ	0.5

Table 2: Parameter values for the numerical exercise.

distribution in the interval $(\underline{\delta}, \bar{\delta}) = (0.05, 1.95)$. In the solution, increasing the amount the firm can gamble (for example, by passing a superpriority law) reduces the amount the firm can borrow, which also reduces the value of the firm's equity.

Figure 5 displays the equity value per unit of capital as a function of cash surplus, and is representative of the overall effect of gambling on firm's value. Compared to no gambling (black curve), gambling without superpriority (blue curve) does not affect equity value significantly, and can even increase it (for different parameter values than those used to create Figure 5), because allowing gambling may enable more gambling for redemption

[22]The multi-period model is solved numerically by iterating the Bellman equations on a discrete set of values of the argument, taking advantage of some features of the solution, for example, that the continuation value asymptotically increases one-for-one with the surplus when the surplus is large, and the firm fails for lack of funding when net surplus is less than some critical value.

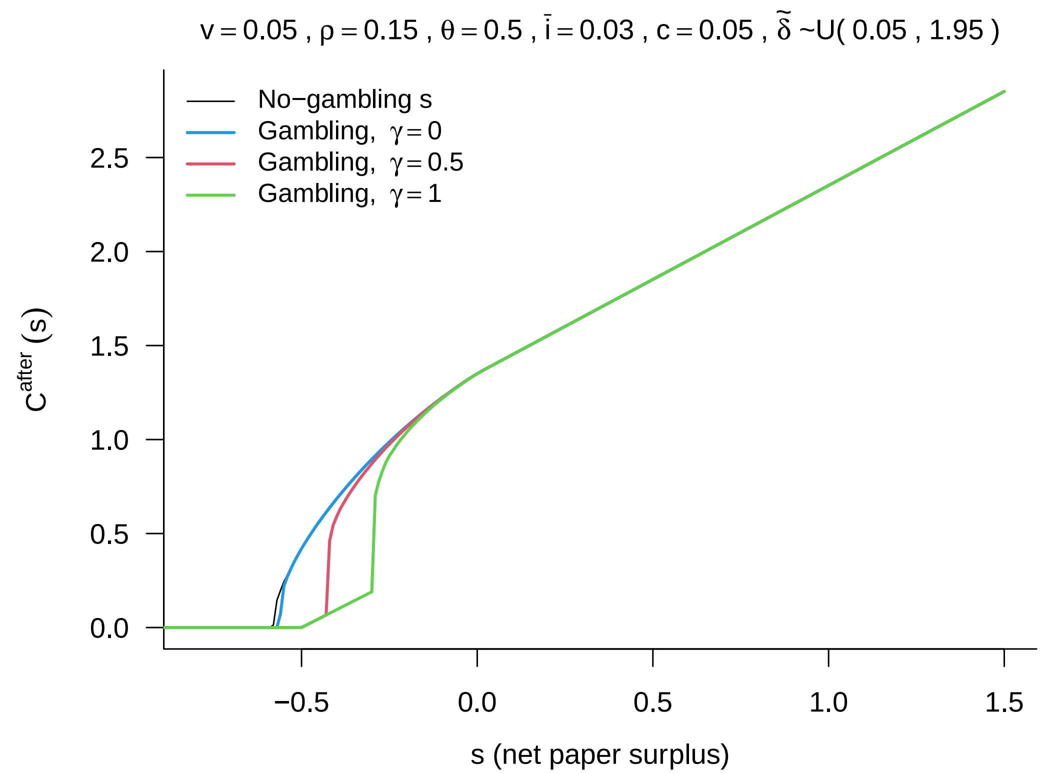

Figure 5: Equity value function (equity continuation value per unit of capital as a function of net surplus per unit of capital). On the horizontal axis, $s = v - \phi + g$ is the surplus per unit of capital after gambling; where surplus is the cash minus face value of debt, whether or not debt will be paid in full. Increasing γ implies that more assets are available for gambling, which reduces equity value. This is because lenders are willing to lend less since they anticipate gambling for ripoff in more contingencies.

than gambling for ripoff.

However, increasing the extent γ of gambling capital, e.g. with superpriority, significantly reduces equity value. This is due to two factors: (1) more can be gambled, and (2) continuation value is diluted because assets that were the basis of continuation value are diverted for gambling. The near-vertical part on each curve gives the smallest surplus for which the firm can borrow enough to avoid failing. The larger is γ, the more the firm can gamble, the less the lenders are willing to lend, and the more often the firm fails. The linear slope of the green curve between about $s = -0.5$ to $s = -0.3$ in Figure 5 has forced liquidation: bondholders' anticipation of gambling for ripoff implies that the firm is unable to borrow enough to pay debt, even though the firm's liquidation value exceeds what is owed.

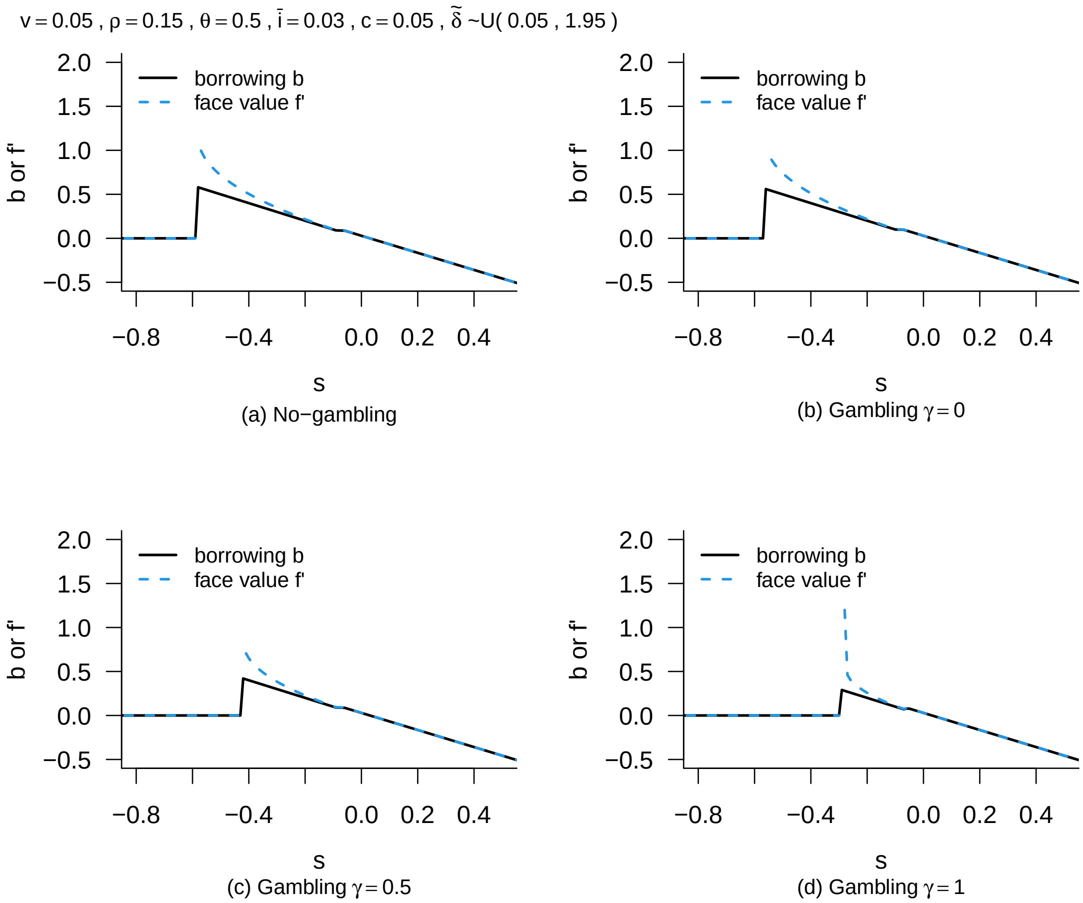

Figure 6: Equilibrium borrowing. The solid curves and dashed curves represent the normalized optimal borrowing and face value of the debt, respectively when the net cash surplus changes. Graphs (a) - (d) show optimal borrowings given different γ's, the amount of capital can be used for gambling. In each graph, as the net cash surplus becomes more negative, borrowing increases to a point to cover the shortfall, with face value increasing disproportionately because of the increased risk. If the shortfall is too large, say, when $s = -0.8$ in each graph, the owners cannot borrow enough and the firm enters bankruptcy. Alternaltively, when s is big, there is a cash surplus, net borrowing can be negative since growth rate is capped by $\bar{i}$, and there can be extra cash holding. When superpriority is introduced, as in graphs (b), (c), and (d), debt becomes even riskier, particularly when the firm can gamble away more assets, as shown in panels (c) and (d), and the firm can borrow less.

The impact of γ on ability to borrow is shown more clearly in Figure 6. The solid and dashed lines in the graphs indicate the normalized optimal debt market value and face value, respectively, as functions of net cash surplus.[23] Graph (a) demonstrates bond pricing when there is no gambling, and (b), (c), and (d) illustrate how varying the proportion γ of capital can be used for gambling impacts bond pricing. When surplus is sufficiently negative, the firm cannot borrow enough to stay alive, and borrowing is zero. The critical surplus where borrowing jumps to zero is the smallest amount of surplus for which the firm can borrow enough to stay alive. Increasing γ increases the critical surplus (meaning survival is less frequent) and decreases maximum borrowing (debt capacity), which equals is the negative of the critical surplus. On the other hand, with significant net cash surplus, investment in capital is at the maximum rate $\bar{\iota}$, resulting in a cash holding denoted by negative borrowing.

Figure 7 illustrates optimal gambling, which can be found following the same procedure as in the single-period model, that is, by "concavifying" the value function $C(s)$ for each contingency. The x-axis gives either after-gambling net cash surplus $s = v - \phi + g$, as s is defined in Section 3, or before-gambling net cash surpus $s = v - \phi$, which is the same as s if $g = 0$. The y-axis gives either the value function after gambling $C^{after}(s)$, or the cash surplus before gambling $C^{b4}(\phi) = C^{b4}(v - s)$. On the range of feasible gambles, the value function $C^{b4}(v-s)$ is the concavification of $C^{after}(s)$. The dashed lines give the value function before gambling. The relevant dashed line has an intercept equal to s, less the maximum amount that can be gambled $-\min(\mathcal{O}_\phi)$, which is the sums of cash from operations v, cash from savings ϕ^-, and maximum gambling from capital $\gamma\theta$. The dashed lines with the largest slopes are similar to gambling for redemption, and the dashed lines with the lowest slopes have gambling for ripoff. The single-period model had only the two extreme cases of gambling for redemption or ripoff, while the multi-period model in this Section is richer and has intermediate cases. Optimal gambling always has two outcomes, which implies that it can be implemented using a digital option, as in the Examples 1 and 2 in the single-period model.

[23]In Figure 6, debt market value and face value are normalized by capital after investment.

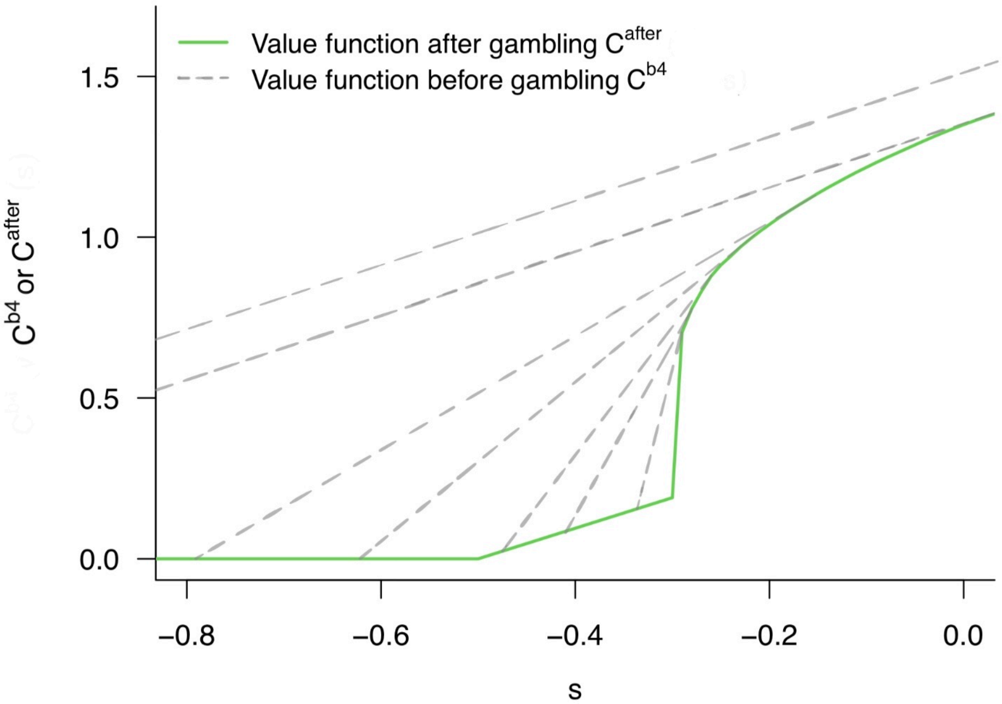

Figure 7: An example of gambling. Gambling has a mixed feature of "gambling for redemption" and "gambling for ripoff" and is continuous rather than jumping between extremes. The different dashed lines give the convexified value before gambling for different maximum amounts that can be gambled ($-\min(\mathscr{O}_\phi)$). More superpriority (high γ) implies more can be gambled, and we have a higher dashed line. This is good myopically for the equity holders, but not once anticipated by bondholders.

5 Conclusions

We provided a simple framework to analyze gambling by firms. "Gambling for redemption" is a Pareto improvement and occurs when the firm owners are eager to continue the firm, whereas "gambling for ripoff" can be socially costly and occurs when continuing a firm is beneficial socially but not to the owners. By making gambling some of the assets possible, superpriority law lowers the value lost to owners in bankruptcy and increases the incentives for the firm owners to gamble for ripoff. In the more realistic intertemporal model with endogenous borrowing and endogenous continuation value, the owners' choices of gambling are intermediate between gambling for redemption and ripoff. We find that superpriority increases the scale of gambling taken by the owners and makes funding more difficult. Our results suggest an interesting empirical question: how do we distinguish "gambling for redemption" and "gambling for ripoff" ex post since they both wipe out the firm's assets in the case of failure? To know the type of gambling, we can instead look at their bets in place, compare the amount of bets stood to gain with and the amount of maturing debt.

One possible implication of superpriority law is the adoption of strategies that reduce the scale of superpriority gambling. On strategy could involve a shift towards more defensive measures that protect against superpriority claims. For example, this shift may involve a preference for bonds protected by perfected collateral, as opposed to relying solely on passive covenants that preclude asset sales and security transfers. Additionally, convertible debt might be as an effective tool to reduce the incentive for engaging in gambling for ripoff, since conversion reduces the face value of debt and increases the continuation value. This is particularly relevant in situations where debt levels are high and equity values are low. Furthermore, firms may choose to use operating leverage strategically to limit the amount of liquidation value available for superpriority gambling. For instance, a company that leases its headquarters building would be less able to gamble its value, compared to a similar company that owns its headquarters building and can potentially exploit superpriority claims to gamble its value.

In this paper, we only scratched the surface of implications of superpriority of QFCs for strategic behavior, focusing narrowly on firms engaging with a single derivatives counterparty. However, adopting strategies that involve multiple counterparties can lead to an asset grab race that undermines the intent of bankruptcy law to ensure an orderly, fair, and efficient outcome. The consequent impact on firm financing and overall efficiency remains to be explored.

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A Proof of Optimal Gambling: redemption

Given constants $F, B, C, \pi \in \mathbb{R}_{++}$, $L \in \mathbb{R}_{+}$, $\bar{G} \geq F - B - \pi$ and $\gamma \equiv \begin{cases} 1, \text{ with superpriority} \\ 0, \text{ absent superpriority,} \end{cases}$ the question becomes

$$\max_{\mathbf{G}(x)} \mathrm{E}\Big[(\pi + \mathbf{G}(\tilde{x}) \geq F - B)(\pi + C + \mathbf{G}(\tilde{x}) - F)\Big]$$
$$\text{s.t. } \mathrm{E}[\mathbf{G}(\tilde{x})] = 0, \text{ and } -\gamma L - \pi \leq \mathbf{G}(\tilde{x}) \leq \bar{G}$$

Since $\tilde{x}$ is the underlying randomness: $\tilde{x} \sim_d U(0,1)$, then w.l.o.g. we assume that $\mathbf{G}(x)$ is non-increasing in x. To get the necessary conditions for the solution, we first concavify the function

$$(\pi + \mathbf{G}(\tilde{x}) \geq F - B)(\pi + C + \mathbf{G}(\tilde{x}) - F) \qquad (*)$$

to make it continuous.

Gambling for redemption: When $F < C - \gamma L$, define $H(\mathbf{G}(x))$ as the concavified function of (*)

$$H(\mathbf{G}(x)) \equiv \begin{cases} \frac{C-B}{F-B+\gamma L}(\pi + \mathbf{G}(x) + \gamma L), & \text{for } \pi + \mathbf{G}(x) < F - B \\ C + \pi + \mathbf{G}(x) - F, & \text{for } \pi + \mathbf{G}(x) \geq F - B \end{cases}$$

The subgradient of $H(\mathbf{G})$ is

$$\nabla H(\mathbf{G}) = \begin{cases} (-\infty, 1], & \text{for } \mathbf{G} = \bar{G} \\ \{1\}, & \text{for } F - B - \pi < \mathbf{G} < \bar{G} \\ [1, \frac{C-B}{F-B+\gamma L}], & \text{for } \mathbf{G} = F - B - \pi \\ \{\frac{C-B}{F-B+\gamma L}\}, & \text{for } -\pi - \gamma L < \mathbf{G} < F - B - \pi \\ [\frac{C-B}{F-B+\gamma L}, +\infty], & \text{for } \mathbf{G} = -\pi - \gamma L \end{cases}$$

Assume $\lambda, w_1, w_2 \geq 0$, and the first order condition of the problem is

$$\lambda - w_1 + w_2 \in \nabla H(\mathbf{G})$$

with

$$w_1 \geq 0, \ (\mathbf{G} + \pi + \gamma L) w_1 = 0$$
$$w_2 \geq 0, \ (\mathbf{G} - \bar{G}) w_2 = 0$$

We ignore the case when $\mathbf{G} \in (-\pi - \gamma L, F - B - \pi)$ since it has measure zero and is not on the original function. We then have

$$\mathbf{G} = \begin{cases} \bar{G}, & \text{for } -\infty < \lambda - w_1 + w_2 \leq 1 \\ [F - B - \pi, \bar{G}], & \text{for } \lambda - w_1 + w_2 = 1 \\ F - B - \pi, & \text{for } 1 \leq \lambda - w_1 + w_2 \leq \frac{C-B}{F-B+\gamma L} \\ -\pi - \gamma L, & \text{for } \frac{C-B}{F-B+\gamma L} \leq \lambda - w_1 + w_2 \leq +\infty \end{cases}$$

(1) If $\pi < F - B$, $\mathbf{G}(x) = -\pi - \gamma L$ is the only $\mathbf{G}(x)$ that is smaller than 0. For $\mathrm{E}[\mathbf{G}(x)] = 0$, there must be some x such that $\mathbf{G}(x) = -\pi - \gamma$. Therefore, $\lambda - w_1 + w_2 \geq \frac{C}{F-B+\gamma L} > 1$ since

$C-\gamma L > F-B$. This implies that $\lambda - w_1 + w_2 = \frac{C-B}{F-B+\gamma L}$ and $\mathbf{G}(x) = F-B-\pi$ or $\mathbf{G}(x) = -\pi-\gamma L$. Thus, by solving

$$0 = \int_{x=0}^{t} (F-B-\pi)dx + \int_{x=t}^{1} (-\pi-\gamma L)dx,$$

we have $t = \frac{\pi+\gamma L}{F-B+\gamma L}$. The optimal gambling is

$$\mathbf{G}^*(x) = \begin{cases} F-B-\pi, & \text{for } 0 < x \leq \frac{\pi+\gamma L}{F-B+\gamma L} \\ -\gamma L-\pi, & \text{for } \frac{\pi+\gamma L}{F-B+\gamma L} < x < 1. \end{cases}$$

Since $\mathbf{G}^*(x)$ solves the concavified problem and is also feasible for the original problem, and the concavified objective function is greater than the original function, we can conclude that $\mathbf{G}^*(x)$ also solves the original problem. If we relax the condition that $\mathbf{G}$ is decreasing, then any gamble with the same distribution would also be optimal.

(2) If $\pi > F-B$, then we must have that for some x, $\mathbf{G}(x) \geq F-B$ and $\lambda - w_1 + w_2 \geq 1$. Then any $\mathbf{G}(x) \in [F-B-\pi, \bar{G}]$ that satisfies

$$\int_{x=0}^{1} \mathbf{G}(x)dx = 0$$

would be a possible solution.

Now we prove that these candidate solutions are the actual solutions. For any candidate solutions $\{\mathbf{G}^*(x) | \pi + \mathbf{G}^*(x) \geq F-B \text{ and } \mathrm{E}[\mathbf{G}^*(x)] = 0\}$,

$$\mathrm{E}[H(\mathbf{G}^*(x))] = C+\pi-F.$$

Since for any feasible solutions $\mathrm{E}[H(\mathbf{G}(x))] \leq C+\pi-F = \mathrm{E}[H(\mathbf{G}^*(x))]$, the candidate solutions are the actual solutions. For the same argument as above, the solutions for the concavified problem are also the solutions for the original problem.

B Proof of Optimal Gambling: ripoff

Gambling for ripoff: When $F > C - \gamma L$, similarly define $H(\mathbf{G}(x))$ as the concavified function of (*)

$$H(\mathbf{G}(x)) \equiv \frac{\pi + \bar{G} - F + C}{\pi + \bar{G} + \gamma L}(\pi + \mathbf{G} + \gamma L)$$

The subgradient of $H(\mathbf{G})$ is

$$\nabla H(\mathbf{G}) = \begin{cases} (-\infty, \frac{\pi+\bar{G}-F+C}{\pi+\bar{G}+\gamma L}], & \text{for } \mathbf{G} = \bar{G} \\ \frac{\pi+\bar{G}-F+C}{\pi+\bar{G}+\gamma L}, & \text{for } -\pi - \gamma L < \mathbf{G} < \bar{G} \\ [\frac{\pi+\bar{G}-F+C}{\pi+\bar{G}+\gamma L}, +\infty), & \text{for } \mathbf{G} = -\pi - \gamma L \end{cases}$$

The first order condition is the same as before. Ignoring the case in which $-\pi - \gamma L < \mathbf{G} < \bar{G}$ since the measure is zero, we have

$$\mathbf{G} = \begin{cases} \bar{G}, & \text{for } \lambda - w_1 + w_2 \leq \frac{\pi+\bar{G}-F+C}{\pi+\bar{G}+\gamma L} \\ -\pi - \gamma L, & \text{for } \lambda - w_1 + w_2 \geq \frac{\pi+\bar{G}-F+C}{\pi+\bar{G}+\gamma L} \end{cases}$$

Therefore, the Lagrange multipliers satisfy $\lambda - w_1 + w_2 = \frac{\pi+\bar{G}-F+C}{\pi+\bar{G}+\gamma L}$. By solving

$$0 = \int_{x=0}^{t} \bar{G} dx + \int_{x=t}^{1} (-\pi - \gamma L) dx,$$

we have $t = \frac{\pi+\gamma L}{\bar{G}+\pi+\gamma L}$. The optimal gambling is

$$\mathbf{G}^*(x) = \begin{cases} \bar{G}, & \text{for } 0 < x \leq \frac{\pi+\gamma L}{\bar{G}+\pi+\gamma L} \\ -\gamma L - \pi, & \text{for } \frac{\pi+\gamma L}{\bar{G}+\pi+\gamma L} < x < 1 \end{cases}$$

For the same argument as above, the solutions for the concavified problem are also the solutions for the original problem.