Limited Liability: Playing It Safe or Going Big? *

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ABSTRACT

Does limited liability on damages improve social efficiency? I show that optimal liability rules trades off tort damages against benefits to outside stakeholders. Full liability promotes care but raises marginal costs, inducing less-than-efficient scale. Limited liability enhances scale but reduces care, proving more efficient than full liability when outside stakeholder value is high. As market competition grows, liability's impact on scale diminishes, and internalizing more damages would increase efficiency. I conclude that limited liability is not one-size-fits-all; tailored policies like requiring insurance for contractors and nuclear decommissioning trusts (NDTs) can help adjust for cross-firm differences.

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1 Introduction

For decades, Manville Corporation (formerly Johns-Manville) concealed the adverse health effects of the asbestos in its products. By 1982, the company was facing over 16,500 tort[1] lawsuits and filed for Chapter 11 bankruptcy reorganization to resolve the cases.[2] Like other mass tort litigations involving public corporations, none of Manville's shareholders were held personally liable for the damages caused by the company due to limited liability rule.[3] To clarify, limited liability refers to the legal concept of restricting the amount of liability to the assets of the company, shielding investors from personal liability. Although the corporation established a pool of funds exceeding $2 billion to compensate claimants, the mounting claims soon dried out the pool, resulting in only a 5.1% payout rate for successful claimants as of 2019.

Without limited liability, shareholders would have been more cautious before investing in a high-risk industry and would have had stronger incentives to monitor the firms. This could have potentially prevented corporations from engaging in practices that harm public health in a large scale. Why should shareholders be absolved of any responsibility?

There are various organization forms beyond just corporations that have limited liability. These include limited liability companies (LLCs) and even partnership-like firms (LPs, LLPs and LLLPs).[4] Additionally, there exists a category of individuals with de-facto limited liability, the so-called "judgment-proof" individuals, with limited wealth, are effectively immune to litigation. It is worth-

[1] A tort is a civil wrong other than a breach of contract that causes a claimant to suffer loss or harm, resulting in legal liability for the person who commits the tortious act. The persons sued for a tort and lose usually have to pay "damages" – that is, a sum of money – to the person who they wronged.

[2] Delaney (1992) has a chapter of detailed discussion of the case and the firm's bankruptcy resolution. It had been observed since 1906 that Asbestos, a fibrous mineral material used for insulation, can lead to lung cancer. However, the asbestos manufacturer claimed not to know about the damage before 1964.

[3] Consider, e.g., *A.H. Robins and the Dalkon Shield, Union Carbide and Bhopal, Johns-Manville and asbestos, Exxon and the Valdez oil spill*, and *Dow-Corning and silicone breast implants*. Roe (1986) provides rationales and evidence that public firms subject to mass tort have all filed for Chapter 11 and in all cases, the valuable assets in the firm were untouched. Though "piercing the corporate veil" provides an exception of limited liability, it is more related to closely held corporations and the veil piercing cases are generally rare, involving directors or managers who have decision making power in the firm. Specifically, Ramberg (2011) notes empirical evidence that no public corporation has ever been pierced the corporate veil.

[4] Both limited liability partnerships (LLPs) and limited liability limited partnerships (LLLPs) inherently possess limited liability due to the expansion of related legislative acts in recent years. Typically, limited partnerships (LPs) designate corporations as general partners to safeguard against liability.

less suing such individuals because the potential recovery is often outweighed by the associated legal costs. On the contrary, "deep pocket" investors refer to entities such as institutional investors, big suppliers and high net worth individuals with substantial financial resources.

Limited liability is thought to be good because it serves as a crucial safeguard to mitigate the risks of investing in a volatile market, enabling investors to take on risky projects and start new ventures. However, we must also recognise the potential damages that corporations can cause and the need to hold them accountable for their actions.

We know that the Coase Theorem suggests that without contracting costs, such as payments to lawyers and accountants and information gathering, liability rules are irrelevant as optimal outcome can be achieved through contracting, which internalizes all the profits and damages. Given the inherent costs, the default legal rule should align with the most agreeable contract. If switching to another liability rule is desired, the parties could do so. This achieves minimal social costs. However, as corporate norms prioritize shareholder value maximization, either because of the defined fiduciary duty of the managers or agents being self-interested, damages to unknown third parties, as seen in cases like Manville's, aren't inherently considered and cannot be priced out, making prior liability negotiation challenging. Therefore, legal frameworks and policies should consider the damages to anonymous tort claimants in terms of effective regulation, enforcement and punishment.

Follow the thought, I build a theoretical model featuring (1) liability coming from torts on a third party, (2) outside stakeholders obtaining potential value from the firm, (3) the firm making a two-dimensional choice of care (or safety) and scale (or quantity), and (4) unilateral care, i.e., only firms' decisions matter. The 4th assumption is an approximation if the other side is unaware of the damages or has very small influence on the losses from accidents.

Under these assumptions, we show that optimal liability rules strike a balance between compensations to tort claimants and the advantages for external stakeholders, including consumers, employees, suppliers, communities, and governments. While full liability promotes diligent care, it concurrently elevates marginal costs, leading to a suboptimal scale. In contrast, limited liability

increases scale at the expense of care. It is more efficient than full liability when the value for external stakeholders is large. It's also noteworthy that as market competition intensifies, the influence of liability on scale wanes, allowing liability rules to incentivize care without substantially affecting scale. The findings emphasize that limited liability doesn't adopt a universal approach. Specific policies, such as setting minimum capital requirement, offering contractor insurances, implementing car registration protocols, and establishing Nuclear Decommissioning Trusts (NDTs), can be tailored to address variances across firms.

The primary finding presents a new perspective on liability protection, highlighting the importance of adapting liability rules based on market conditions. Specifically, we propose liability protection for natural monopolies while advocating against it for industries with low market barriers. While the traditional argument for limited liability focuses on reducing risks and encouraging new ventures and investment in unstable markets, our research emphasizes how liability rules can address externalities resulting from the market microstructure.

In Section 2, I start by introducing a straightforward model that assumes both the share of benefits to outside stakeholders and the liability to tort claimants are exogenous fractions. Despite its simplicity, this model provides key insights into the tradeoff between positive and negative externalities and the optimal liability rules. That is, unlimited liability can provide correct incentives for care but under-provision of scale, and limited liability mitigates the scale efficiency but sacrifices provision of care.

In Section 3, I examine how the externalization of benefits can be endogenous by introducing market power. In this setting, outside stakeholders are represented by consumers who obtain surplus from the firm. With market power, firms can influence the externalization of benefits and damages from productivity. Specifically, larger scales can increase the fraction of benefits to outside stakeholders, such as lower crime rates and increased job creation for communities, higher tax revenues for governments, and larger consumer surplus for consumers.[5] Meanwhile, higher levels of care, such as monitoring activities and developing safer technology, can reduce the frequency

[5]If we suppose that the demand for each firm is not perfectly elastic and the firm does not price discriminate.

and probably the severity of damages to tort claimants. In addiction to the conclusion in Section 2, we claim that when demand is highly inelastic, the large proportion of consumer surplus results in inefficiencies in scale, which makes limited liability more socially appealing compared to unlimited liability.

Section 4 extends the model to multiple firms engaging in Cournot competition. I assume that the firms are homogeneous and invest in the same technology using identical resources. With more competition, I demonstrate that full liability would eventually converge to the first-best and limited liability would result in over-investment in scale and under-investment in care. This is essentially because competition inherently ensures efficient scaling. As competition intensifies, concerns regarding scale diminish since competitive forces dictate it. Any alterations in scale by a single entity have a negligible impact on the broader industry scale. Therefore, in a highly competitive environment with numerous firms, the industry scale remains close to optimal when the firm changes case level. The primary role of the liability rule then is to incentivize proper care.

Section 5 compares the effects when capital has different ownership, that is, capital purchased by the owners versus capital financed by non-recourse debt. With capital financed by non-recourse debt, the tort claimants can only be paid with the firm's proceeds, but not the other assets. However, purchased capital is part of the firm's assets and can be used as payments to the tort claimants. As stated, when market is more competitive, firms under limited liability would over-invest in scale and under-invest in care, but if the tort claimants are in a higher priority order to receive more assets in compensation, there is reduced inefficiency in care.

The model can also be applied to other markets as discussed in Section 6. An analogous market is the labor market where employees are the stakeholders. When a firm is the only buyer of labor in the market, the firm has monopsony power. The marginal cost of employing workers would be greater than the social cost which is also workers' reservation utility. Because of this, the firm does not capture full benefits from the employees working in the firm. Because of this, the firm does not capture full benefits from the employees working in the firm. A classic example would be a coal mining company in a geographically remote areas such as the West Virginia where finding

substitutions of a job is costly. Parallel to the product market, more liability on torts results in higher care. On the extensive margin, higher care level as well as fixed costs may reduce scale to zero. On the intensive margin, higher care level increases the marginal cost of production and decreases scale. Limited liability on accidents would induce higher labor employed by the firm compared to full liability and is efficient if the welfare gain from the increased employment and the operation of business covers the loss from lower investment in care. However, having a lot of competitors is where limited liability is inefficient, and similarly, the inefficiency can be lowered by requiring some capital installed in the firm as a cushion for tort claimants. In this section, governments and communities as stakeholders are also discussed.

1.1 Tort: our Focus

When discussing tort claims, it's essential to recognize pollution does not apply. While pollution might seem synonymous with tort claims, given that it inflicts damage on third parties, the challenge lies in identifying the affected parties. For instance, consider Chicago, where 800 companies emit particulates. When someone contracts emphysema, a condition linked to particulates, pinpointing the culpable company becomes complex. The individual's condition might not be traceable to any specific company, or even if it's due to particulate exposure at all. Class action suits could be a recourse, but these situations are more akin to public goods issues and require distinct mechanisms a address.

Instead, we focus more on the product tort and liability. Consider delivery companies like FedEx or UPS. If they urge their drivers to hasten deliveries, they might save costs on drivers and vehicles. However, this haste could lead to more accidents, thereby increasing tort claims.

Another example could be vendors selling vegetables, such as sponge gourds, a vegetable that can contain cucurbitacins when mature. If a vendor doesn't inspect each gourd thoroughly or if restaurants don't clean them properly, consumers might face food poisoning risks, leading to potential tort claims.

1.2 Implement Liability Rules

As we mentioned before, even with unlimited liability, if the investor is judgment-proof, which means that the investor has zero available assets to seize, the court has no rights to seize the assets.

That is to say, a *de facto* full liability can hardly be achieved directly given the existing laws for corporations, LLCs and special forms of partnerships. Plus, changing the regime of the law is costly, and the optimal liability rules shift when the market conditions change, which makes it meaningless to have a enormous change of the default rule. However, limited liability in combination with other policies are in effect increasing liability, for example, minimum capital requirements, requiring insurance, and setting up funds.

Minimum capital requirements often refer to standardized regulations for banks and other depository institutions to include a minimum amount of liquid assets against their risky assets.[6] In the United States, capital requirements on partnerships, corporations and LLCs are only imposed by contracting between the firms and lenders or other third parties rather than by law. In many other countries, corporate laws require some organizations to hold minimum assets, and the primary purpose was "to protect creditors and nurture confidence in financial markets."[7] For example, all public firms in European Union have to hold capital of value at least €25,000,[8] and in the United Kingdom (England and Wales) the amount required is £50,000.[9]

Requiring insurance is relatively easy when bonded with other regulation. Mandating insurance emerges as a pragmatic approach to calibrate liability levels. In this framework, while tort claimants bear a portion of the claim, insurance companies absorb another segment. This latter portion is ultimately absorbed by the firm, especially with a competitive insurance market.

For example, in the US most of the states require proof of insurance with car registration and only 7 states have exceptions. The rationale is straightforward: if an individual inflicts in-

[6] Haubrich (2020)

[7] Bank (2013)

[8] https://www.dlapiperintelligence.com/goingglobal/corporate/index.html?t=03-minimum-capital-requirement

[9] https://www.dlapiperrealworld.com/law/index.html?t=corporate-vehicles&c=GB-ENG-WLS&s=setting-up-a-corporate-vehicle&q=minimum-capital

jury, whether negligently or intentionally, the victim's medical expenses are guaranteed coverage. The imposition of this insurance mandate stems from a pragmatic understanding that a significant proportion of the populace lacks the financial resources ("deep pockets") to compensate victims directly. Without such a mandate, many victims would remain uncompensated, bearing the brunt of another's negligence. This principle of protecting against "litigation-proof" entities due to financial constraints parallels the notion of limited liability.

Consider the case of contractors, often sole proprietors, who might not be affluent but offer services like roof repairs. Many jurisdictions necessitate such workers to not only be insured (covering potential damages or injuries during work) but also to be bonded. Bonding ensures that if a contractor abandons a project midway, the bonding company steps in to finance the completion of the job, preventing homeowners from being stranded with unfinished projects. The essence of such regulations is to ensure accountability. While many contractors might inherently possess limited liability, not due to their business structure but because they are "litigation-proof" from limited assets. The underlying premise is clear: while business structures might offer limited liability, regulatory frameworks can and should ensure that consumer interests aren't compromised.

Setting up funds are common for entities involved in environmental contamination.[10] The financing of the funds varies. For example, The Oil Spill Liability Trust Fund (OSLTF) in the US mainly collects barrel tax on the petroleum industry, and the Nuclear Decommissioning Trusts (NDTs) can be a combination of prepayments, sinking funds,[11] and insurances or even guarantees from parent companies. This is to say, it is relatively simple to change the amount of insurance and funds required, and consequently firms can bear liability.

Of course, there are some other liability rules that can be applied. When we merge care standards with limited liability, the airline industry provides a pertinent case. Airlines have very strict rules for checking and maintaining their planes, since if there's an accident, the company's responsibility might not be enough to cover all the damages. This is even more important for airlines that

[10] Bayon et al. (1999).

[11] A prepayment is a deposit by the licensee at the start of operation, similar to capital installation. A sinking fund is the account to accumulate funds set aside by the firm over time.

are financially constrained. If they neglect repairs, the risks skyrocket. Insurance, in such cases, can indirectly enforce care standards.

1.3 Legal Considerations and Literature

While the paper has a law flavor, it is worth mentioning two notions of fairness that are often considered in the law literature on injuries. The first notion of fairness suggests that injurers should be held accountable for the harm they have caused, even if their actions were not wrongful, a form of ”an eye for an eye, and a tooth for a tooth.” The second notion concerns compensation that makes the victims whole and is justified by Coleman (1995) on the grounds of a logic of corrective justice: “If A wrongfully injures B, A must pay B for the loss B suffers as a consequence of A’s act.” This aligns with the Common law tort doctrine.

Following these two notions, many previous law literature conclude that investors should internalize all tort risks. For example, Hansmann and Kraakman (1990) claims that “there may be no persuasive reasons to prefer limited liability over a regime of unlimited pro rata shareholder liability for corporate torts.” LoPucki (1994) also proposes the abolition of limited liability to achieve “the goal of minimizing the externalization of tort liability.” In our broader framework that also considers scale efficiency, internalizing all the damages can only be justified if the firm does not have positive externalities, because the firm would align the interests with society – only a special case in this paper.

While it would be interesting to discuss the philosophy of these two notions, my paper primarily focuses on the economic efficiency aspect, given that we want firms to conduct proper care while still encouraging innovation and scale.

The literature in favor of limited liability can be categorized into three main arguments. The first argument is that limited liability is necessary to secure capital for risky projects or new ventures when risk-averse investors overreact to negative returns or are hesitant to invest due to moral concerns about environmental damage. The second argument suggests that limited liability is preferable to full liability because it incentivizes potential tort claimants or other stakeholders to

take precautions by externalizing some damages to them. This bilateral care framework has been studied by Landes and Posner (1985) and Hay and Spier (2005). Finally, the third argument proposes that unlimited liability is impractical due to (1) the transaction costs associated with collecting fees, especially for public corporations, and (2) the potential for shareholders to hide assets to evade liabilities. The following paragraphs provide more details about the practicality of unlimited liability and my proposed resolution.

Unlimited liability says that unmet obligations of the firm are obligations of the owner, and is always bounded by the debtor's wealth unless one can take a penalty of a pound of flesh as in *The Merchant of Venice* or in a form of indentured servitude or even one's life. All are not accepted in today's civilized world. Even if possible, unlimited liability is still limited more or less. Full liability in the model setting requires both unlimited liability and what is referred to as "deep pockets," namely that the owner has sufficient assets that can be seized in satisfaction of the obligations. If an owner instead has a "small pocket, " the obligations are only partially satisfied.[12]

Therefore, knowing one's personal wealth is important in the realization of unlimited liability. For closely held firms, unlimited liability is usually claimed appropriate because forcing to internalize tort risk has relatively small social costs.[13] Unlimited liability is costly for public-held firms. In particular, one big cost for the joint and several unlimited liability regime is to know the available wealth of other shareholders since the shareholders who have more assets bear more tort liability. This could result in large investors heavily monitoring the firm, or no investing, or even highly leveraged firm with a very large amount of secured debt that has priority over tort liability.[14] A famous paper Hansmann and Kraakman (1990) proposes pro rata unlimited liability over joint and several rule to solve the information and monitoring problems. Bainbridge and Henderson (2016) disagrees on the rule made by Hansmann and Kraakman (1990) to identify responsible persons, suggesting that it is not practical given the fast changes of the firm's shareholder pool especially with today's financial market. Leebron (1991) also argues that the collection costs

[12]"Judgment-proof" is a special case of small pocket. For instance, a family have all their wealth in a family firm is "judgment-proof."

[13]Easterbrook and Fischel (1985).

[14]Grundfest (1992) argues that there would be "more exotic debt-equity hybrid."

would be too high considering possibly hundreds of thousands of shareholders among whom some are out-of-state, some are off-shore, some own too few shares to worth the effort, and some have small pockets. In addition, investors could always take advantage of other law to evade personal liability, e.g., investing in some real estate or employing independent contractors, etc.[15] The investors are judgment-proof if they can effectively evade all the personal liabilities. In papers study judgment-proof investors, Shavell (1986) shows that too much risk and too little care would be taken, and Che and Spier (2008) claims that injurers strategically using senior bonds to judgment proof themselves will result in less precaution. Hansmann and Kraakman (1990) has claimed to solve these problems, for example, by proposing mandatory insurance.

2 The Benchmark Model: exogenous externalities

For now, I focus on investment of care and scale. I consider the simplest case in which there is a single investor, and the externalization of profits and liability are strictly pro rata. Strict pro rata liability means that the liability is proportional to the actual damage regardless of financial constraints. In the next section, liability is more realistically constrained by the assets, and therefore financing is important. The main intuition of investment can be understood in the simplest model, but the model featuring realistic liability rules is convenient for policy discussion, e.g., capital requirement, taxation, and insurance requirements.

In this section, a single investor can choose either to invest in a firm whose product potentially causes damages. The investor decides the allocation of investment into scale Q and cost of care s. The scale of investment generates net benefits $F(Q)$ with $F''(Q) < 0$, and of which a fraction $\mu \in [0,1]$ goes to the investor (proprietor) and $1-\mu$ goes to the outside stakeholders. $s \in [0,1)$ represents the care level and is the probability that no damages occur. The total cost of care is $C(s)Q$, where $C(s)$ can be seen as the unit cost of care that is monotone and convex (i.e., $C'(s) \geq 0$,

[15] In California, money home mortgages are non-recourse. In agency law, if the tort is committed by an independent contractor, the principal would have limited liability. See Bainbridge and Henderson (2016)

$C''(s) > 0$). $C(0) = C'(0) = 0$ and $\lim_{s\uparrow 1} C'(s) = \infty$.[16] The convexity of cost function implies that the marginal cost of care level is increasing in care. The outside option of investment is a safe technology with payoff $R_f \geq 1$ per unit of investment.

There are only two states of the world: a safe state, with probability s, and a tort state, with probability $1-s$. Damages occur only in the tort state with foreseeable total damage Qd which is proportional to scale. The investor bears liability proportional to the damages to tort claimants, namely, λQd, where $\lambda \in [0,1]$. The average unit damage d is a constant in the model. In reality, the higher care level may also reduce the per unit damage. For example, requiring fastening safety belt while driving reduces both frequency and severity of physical damages. I assume that higher care level only reduces the frequency for simplicity.

The socially efficient allocation of resources to scale and care should consider all the benefits and damages, but the investor only considers the fractions that apply to them. We can write down the optimal social value V_s and the investor's value V_f

$$V_s = \max_{s,Q} \; F(Q) - C(s)Q - (1-s)Qd$$

$$V_f = \max_{s,Q} \; \mu F(Q) - C(s)Q - (1-s)\lambda Qd$$

requiring that $s \in [0,1]$ and $Q \geq 0$. Now we assume (s^*, Q^*) is the social optimal investment and (s_f^*, Q_f^*) is the firm's optimal investment, then our results can be illustrated by Figure 1 and Figure 2, and are also stated in Prop 1.

PROPOSITION 1. *Firm's choice of care and scale in relation to liability can be expressed by the following*

1) If the outside stakeholders obtain large enough benefits from the firm, full liability results in less-than-optimal scale ($Q_f^* < Q^*$) and optimal care ($s^* = s_f^*$), which can be very inefficient when μ is small.

[16] The assumption $\lim_{s\uparrow 1} C'(s) = \infty$ is not crucial but to construct an interior solution for safety. This is also reasonable since it is never possible to ensure no accidents.

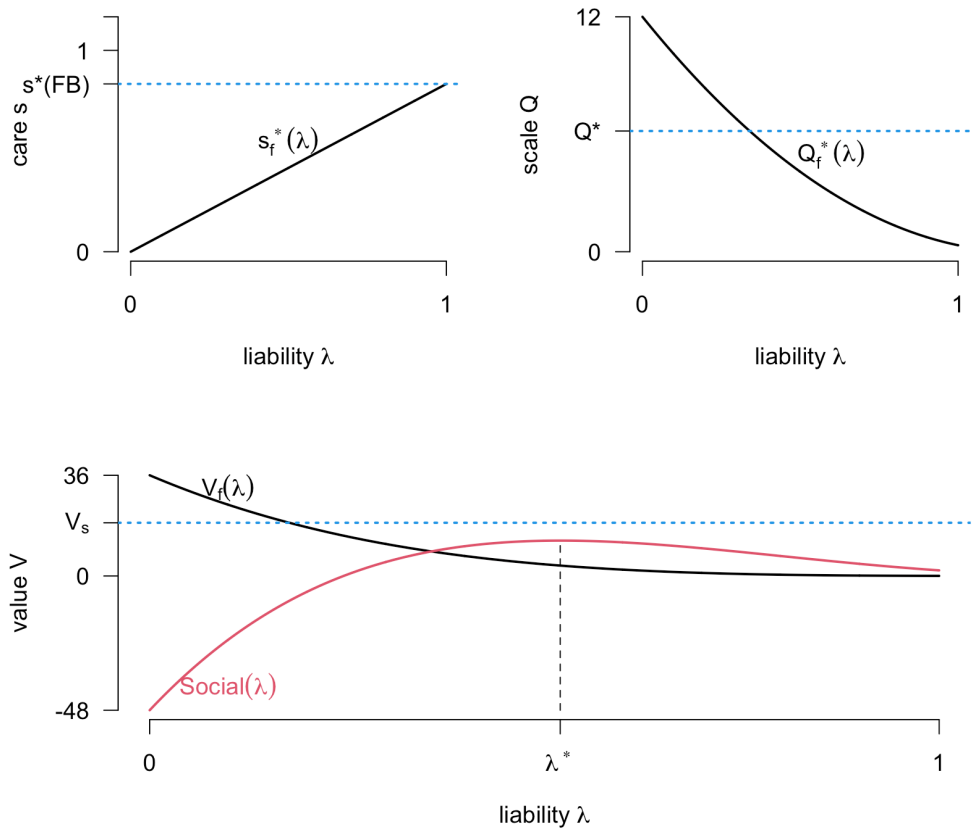


Figure 1: ***(Benchmark: firm's choices of care and scale and firm value):*** *the x-axis is the fraction of liability that the investor bears, and the y-axes are the choices of care and scale of the investor (the top graphs) and the optimal firm value as a function of liability accordingly (the bottom graph). We can also compare the choices and the value to the first best (s^*, Q^*, V_s, respectively, demonstrated by the blue dashed lines). The red curve in the bottom graph represents social value subject to the investor's optimal choice of care and scale. It shows an optimal level of liability* λ^* that results in the second best social outcome.

2) *Less liability results in lower care and larger scale since* $\frac{\partial s_f^*}{\partial \lambda} > 0$ *and* $\frac{\partial Q_f^*}{\partial \lambda} \leq 0$. *Therefore, partial liability can balance the gain from increased scale and the loss from reduced care and hence there is an optimal level of liability.*

3) *Full liability is more appealing when the benefits to the outside stakeholders decrease (*μ *increases). Specifically, when the investor internalizes all the benefits and all the damages (*$\mu = \lambda = 1$*), firm's problem coincides with the social choice, which results in optimal levels of care and scale.*

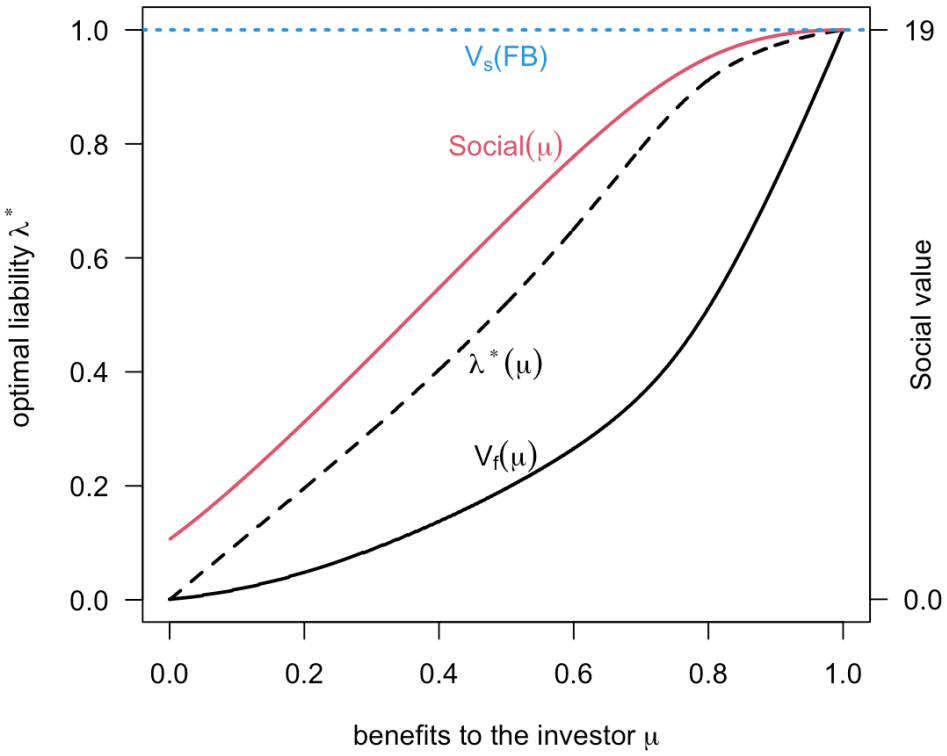


Figure 2: ***(Benchmark: social choice of optimal liability and related firm value and social value):*** *the x-axis is the fraction of benefits to the investor, and the left y-axis represents the optimal liability function that solves the second best ($\lambda^*(\mu)$, the dashed black curve). The solid black curve represents the firm value based on the optimal liability rule, and the solid red curves represents social value. As we can see, as the benefits to the outside stakeholder decrease (as μ increases), the optimal liability for the investor increases, so do both firm and social values. If the firm can internalize all the benefits, say, $\mu = 1$, then full liability is the optimal and results in first best.*

Proof. For interior solutions, the optimal levels of investment can be expressed by the first order conditions:

$$\text{Social} \begin{cases} (s) & C'(s^*)Q^* = Q^* d \\ (Q) & F'(Q^*) = C(s^*) + (1-s^*)d \end{cases}$$

$$\text{Firm} \begin{cases} (s) & C'(s_f^*)Q_f^* = Q_f^* \lambda d \\ (Q) & \mu F'(Q_f^*) = C(s_f^*) + (1-s_f^*)\lambda d \end{cases}$$

Since $\lambda \in [0,1]$, $C''() > 0$, $s^* \geq s_f^*$. We can also do the following computation

$$\frac{\partial s_f^*}{\partial \lambda} = \frac{d}{C''(s_f^*)} > 0$$
$$\frac{\partial Q_f^*}{\partial \lambda} = \frac{(1-s_f^*)d}{\mu F''(Q_f^*)} < 0 \text{ (} \textit{hence proves Prop. 1).}\text{)}$$

The optimal level of liability solves the following problem given μ,

$$\max_{\lambda} F(Q_f(\lambda)) - C(s_f(\lambda))Q_f(\lambda) - (1-s_f(\lambda))Q_f(\lambda)d$$
$$s.t.\ \lambda \geq 0,\ C'(s_f(\lambda)) = \lambda d$$
$$\mu F'(Q_f(\lambda)) = C(s_f(\lambda)) + (1-s_f(\lambda))\lambda$$

solving the F.O.C., we have

$$\begin{aligned} 1-\lambda^* &= (1-\mu)\frac{F'(Q_f)\frac{dQ_f}{d\lambda}/d}{(1-s_f)\frac{dQ_f}{d\lambda} - Q_f\frac{ds_f}{d\lambda}} \\ &= (1-\mu)\frac{(1-s_f)F'(Q_f)C''(Q_f)/d}{(1-s_f)^2C''(s_f) - \mu Q_f F''(Q_f)} \end{aligned}$$

□

3 The Product Market

If the firm can always capture a fixed fraction of benefits, then a fractional input subsidy and fractional liability would result in the first best. However, μ can be endogenously determined. We look at one example of endogenous μ, that is, when the firm has market power. But first, we start with a perfect competitive market so that we can compare it with the more interesting case later.

In this section, the consumers are the only other stakeholders of the monopoly firm selling a potentially dangerous product, and I specifically look into different liability rules. For convenience, the single firm uses rental capital which cannot be grabbed by the tort claimants. The firm

can only use revenue in compensation of the tort claimants. In the following sections, I increase competition by increasing the number of the firms and I also discuss the impact of rental capital versus purchased capital on the investors' choices of care (how much the firm invests in ensuring that the product is safe) and scale (the quantity produced). I will discuss other stakeholders such as employees and government who also capture some of the rents from operation through various institutions.

I focus on a single investor with wealth W who can choose either to invest in a firm or to invest in a safe technology with a payoff $R_f > 0$ (one plus the rate of return) per unit invested.[17] The firm's product potentially causes damages. The investor decides the allocation of investment into a firm with input Q, which produces Q units of goods. The care level or safety of the product is

The investor's firm is a monopolist in the product market and sets price p for each unit of good. The market has an inverse demand function $m(x)$ (satisfying $m'(x) < 0$) which is also the marginal utility of the x^{th} good consumed by the consumers, excluding any damages not compensated. In this paper, I proceed with the pure tort case in which the consumers are not subject to damages, or they act as if they are not subject to damages.[18] We can simply think that the damages are fully borne by a third party who is unknown until the damages occur. Given price p and the inverse demand function $m(x)$, a representative consumer chooses a scale (quantity) $Q \in \mathbb{R}_{\geq 0}$ to maximize consumer surplus

$$V_c = \int_{x=0}^{Q} \Big[m(x) - p \Big] dx \tag{1}$$

In a monopolistic world, In equilibrium, $p = m(Q)$, namely, the price should be equal to the marginal benefit of consuming Q goods. In equilibrium, the total consumer surplus is

$$V_c = \int_{x=0}^{Q} \Big[m(x) - m(Q) \Big] dx. \tag{2}$$

[17] Wealth W is large enough to avoid discussing financial constraint and trade-offs of investments in operation and safety at the boundary, but it is not unlimited so that the investor is possibly small-pocket.

[18] For example, a customer purchasing goods from Amazon cares about the prices and not the probability that they would get hit by the Amazon trucks, or the consideration of accidents is negligible.

Under the limited liability rule, liability is restricted to firm's available assets which is the total revenue from selling the product, i.e. $\Lambda(Q,p,w) = (Qp) \wedge (Qd)$, where $a \wedge b \equiv \min\{a,b\}$ denotes the minimum of a and b, and $a \vee b \equiv \max\{a,b\}$ denotes the maximum of a and b. The effective liability of a deep-pocket investor under unlimited liability rule is full liability Qd, but that of a small-pocket also depends on the appropriable wealth outside of the firm and is $w \equiv [R_f(W-(1+C(s))Q)]$. For an investor with limited wealth, the larger the investment in the firm, the more likely the investor is to be small-pocket.[19]

Given the investor's choices of care and scale, the social value of the project is

$$S(s,Q) = \int_{x=0}^{Q} \Big[m(x) - (1-s)d\Big]dx - R_f(1+C(s))Q. \tag{3}$$

$$\text{s.t.} \quad Q \geq 0 \tag{4}$$

Because we are looking at total surplus, liability is just a transfer from firms to tort claimants and is not in the social value function. With first-best care level s_s^* and scale Q_s^*, the first-best social value is equal to the consumer surplus[20]

$$\begin{aligned} S(s_s^*, Q_s^*) &= \Big(\int_{x=0}^{Q_s^*} \Big[m(x) - (1-s_s^*)d\Big]dx - R_f(1+C(s_s^*))Q_s^* \Big)^+ \\ &= \Big(\int_{x=0}^{Q_s^*} m(x)dx - m(Q_s^*)Q_s^* \Big)^+. \end{aligned} \tag{7}$$

With smaller reservation utility, firms tend to produce more[21] and create larger total damages,

[19] I don't explicitly model the uncertainty in the litigation process, but that can be implicit embedded in s. If the strict liability rules apply, the firm is responsible whenever an accident occurs; if under the negligence laws, there can be proof of burden and there is uncertainty whether the liability is going into litigation (add reference) especially when the tortious technology is new and not well understood even by the experts. Firms could then perform strategically to affect the litigation. It is an interesting topic but here I assume that the expected liability is foreseeable.

[20] The first-best s_s^* and Q_s^* satisfy

$$C'(s_s^*) = \frac{d}{R_f}, \tag{5}$$

$$m(Q_s^*) - (1-s_s^*)d = R_f(1+C(s_s^*)). \tag{6}$$

Equation (7) is derived when replacing the total cost of consumption with $m(Q_s^*)$ (from equation (6)), and the social value equal to the consumer surplus is always non-negative.

[21] $dQ_s^*/dR_f = \frac{1+C(s_s^*)+R_f}{m'(Q_s^*)} < 0$.

so that there are more incentives to choose a higher care level. Similarly, Equation (6) indicates that the risk-neutral investor either will not invest at all when the potential damage is too high or the risk-free investment has a high return, or will keep investing until the marginal return of investment in the firm is exactly equal to the risk-free payoff.

I assume interior solutions so that $w \equiv R_f(W-(1+C(s))Q)$ is strictly positive. The investor's problem is to choose care s and scale Q, followed by the consumer's consumption choice. Formally,

$$\max_{s,Q}\left(Qp-(1-s)\Lambda(Q,p,w)+w\right) \tag{8}$$

$$\text{s.t.} \quad p=m(Q), 0\leq Q \tag{9}$$

$$w \equiv R_f(W-(1+C(s))Q) > 0 \tag{10}$$

$$\text{and} \quad \underline{s} \leq s < 1, \tag{11}$$

where

$$\Lambda(Q,p,w)=\begin{cases} 0, & \text{no liability} \\ (Qp)\wedge(Qd), & \text{limited liability} \\ \Big(Qp+w]\Big)\wedge(Qd) & \text{unlimited liability with small pockets} \\ Qd, & \text{unlimited liability with deep pockets (like } w=\infty) \end{cases}$$

We can see that the optimal choices of care and scale by firm are affected by the externalization of benefit (through consumer surplus) and the internalization of damage (through liability rule). To ensure the existence and nice features of solution, I make the following assumptions:

Assumpt. *A:* $m(Q)$ *is continuous and smooth with* $m(Q)>0, m'(Q)<0$ *and* $\lim_{s\to\infty} m(Q)\leq 0$.

Assumpt. *B:* $m(0)>R_f, m'(0)>-\infty$.

Assumpt. *C:* $d^2[m(Q)Q]/(dQ)^2 = 2m'(Q) + m''(Q)Q < 0$*, or equivalently*

$$2 + \frac{d\log m'(Q)}{d\log Q} > 0. \tag{12}$$

Assumption A says that the demand curve is downward-sloping, and Assumption B is necessary for the project to be socially valuable because consuming the very first unit should generate more than the reservation payoff. Assumption C ensures unique solution in most cases and will be used in the next section. The following Table 1 compares the investor's choices with different liability rules to the first-best. For more detailed computation see Table 2.

Rules	s^*	Q^*	Q_M^*
FL	$=$	$<$	$<$
LL	$<$	$\lessgtr$	$<$
UL-SP	$\lessgtr$	$\lessgtr$	$<$
NL	$<$	$\lessgtr$	$=$

Table 1: ***(Comparison of different liability rules to first-best and pure monopoly)*** FL: full liability (unlimited liability + deep-pocket investors); LL: limited liability; UL-SP: unlimited liability + small pockets; NL: no liability; (s^*, Q^*): first-best safety and quantity; Q_M^*: equilibrium quantity when safety has zero cost.

PROPOSITION 2. *With full liability, investment in care achieves first-best but there is always under-investment in scale because the investor does not internalize all the benefits. Limited liability under-provides care but increases investment in scale compared to full liability. This can be socially harmful when demand is very elastic and potential tort liability is large.*

Proof. With full liability, the first order conditions are

$$(s) \quad C'(s_u^*) = \frac{d}{R_f} \Rightarrow q_u^* = q_s^*$$
$$(Q) \quad \frac{m(Q_u^*) + m'(Q_u^*)Q_u^* - (1 - s_u^*)d}{1 + C(s_u^*)} = R_f$$

(1)	(2)	(3)	(4)	(5)	(6)	(7)
rule	$\Lambda(Q,p,w)$	FOC:(s)	FOC:(Q)	Firm started ($Q^*>0$)	$s^*<s_s^*$	$Q^*>Q_s^*$
FB	-	$C'(s_s^*)=\frac{d}{R_f}$	$m(Q_s^*)\leq R_f\left(1+C(s_s^*)\right)+(1-s_s^*)d$	$m(0)>R_f\left(1+C(s_s^*)\right)+(1-s_s^*)d$	(always $=$)	(always $=$)
FL	Qd	$C'(s_u^*)=\frac{d}{R_f}$	$m(Q_u^*)+m'(Q_u^*)Q_u^*\leq R_f\left(1+C(s_u^*)\right)+(1-s_u^*)d$	$m(0)>R_f\left(1+C(s_u^*)\right)$	(always $=$)	(always $<$)
LL	$Qp(<Qd)$	$C'(s_l^*)=\frac{m(Q_l^*)}{R_f}$	$m(Q_l^*)+m'(Q_l^*)Q_l^*\leq \frac{R_f\left(1+C(s_l^*)\right)}{s_l^*}$	$m(0)>\frac{R_f\left(1+C(s_l^*)\right)}{s_l^*}$	(always $<$)	$e_p(Q_l^*)>\left[1-\frac{R_f\left(1+C(s_l^*)\right)}{s_l^*m(Q_s^*)}\right]^{-1}$
UL-SP	$Qp+w$ $(<Qd)$	$C'(s_j^*)=\frac{m(Q_j^*)+R_f(W/Q-1-C(s_j^*))}{s_j^*R_f}$	$m(Q_j^*)+m'(Q_j^*)Q_j^*\leq R_f\left(1+C(s_j^*)\right)$	$m(0)>R_f\left(1+C(s_j^*)\right)$	(can be $\lessgtr$)	$e_p(Q_j^*)>\left[1-\frac{R_f\left(1+C(s_j^*)\right)}{m(Q_s^*)}\right]^{-1}$
NL	0	$C'(s_0^*)=0$	$m(Q_0^*)+m'(Q_0^*)Q_0^*\leq R_f$	$m(0)>R_f$	(always $<$)	$e_p(Q_0^*)>\left[1-\frac{R_f}{m(Q_s^*)}\right]^{-1}$

$e_p\equiv-\frac{dI/I}{dp/p}=-\frac{m(Q)}{m'(Q)Q}$

Table 2: ***(Liability rules and solutions)*** When the firm has market power and demand is not perfectly elastic, full liability (FL) or unlimited liability with deep-pocket investors (UL) always results in first-best care but lower scale; the firm with limited liability has less incentives for care but more incentives in scale compared to FL. However, when the firm has unlimited liability, with small pockets (UL-SP) there can be under- and over-investment in both care and scale depending on the parameter values. With no liability (NL), the firm invests minimum in care and there can be under- and over-investment in scale depending on the demand elasticity.

Compare the second equation to Equation (6),

$$m(Q_s^*) = m(Q_u^*) + m'(Q_u^*)Q_u^*$$

Since $m'(Q_u^*)Q_u^* < 0$, it must be true that $m(Q_s^*) < m(Q_u^*)$, so that $Q_s^* > Q_u^*$. Similarly, we can prove the proposition with limited liability by comparing the first order conditions. See Appendix A for details. □

Unlimited liability with deep-pocket investors can undermine the incentives for scale, and the inefficiency can be huge when demand is less elastic and the consumer surplus is large. Limited liability may generate slightly less incentives for safety, but the inefficiency can be compensated by higher scale and result in higher social value. Limited liability can be regarded as a subsidy from the tort claimants to the investor and higher quantity compared to full liability. The incentives for safety is dampened due to the externalization of tort damages, which also mitigates the externalization of benefits and encourages bigger scale. This can be good when too much benefits are externalized due to inelastic demand. However, limited liability is not a one-size-fits-all rule, firstly because there can be over-compensation when the tort claims are large and when demand is very elastic, and secondly, when the investor is small-pocket, the liability is capped even with unlimited liability.

PROPOSITION 3. ***(Unlimited liability with small pockets)*** *If* $\Lambda(Q,p,w) = Qp + w < Qd$,[22]

(1) Compared to first-best, there can be under-provision or over-provision of both safety and quantity.

(2) Compared to limited liability, if $Q_j^* < Q_l^*$*, then* $s_j^* > s_l^*$.

(3) The investor is more likely to have small pockets with larger damage d.

[22]The investor being small-pocket or deep-pocket is endogenous. The original firm's problem is not smooth at $\Lambda(Q_f,p,w)/Q_f = d$, since the gradients are different when approaching $\Lambda(Q_f,p,w)/Q_f = d$ on different sides. To solve such a problem I obtain the optimal solution on each side (i.e., conditional on whether $\Lambda(Q_f,p,w)/Q_f$ is greater than d or not) and compare them to get the global optimum. If conditional on $\Lambda(Q_f,p,w)/Q_f < d$ but instead the interior solution satisfies that $\Lambda(Q_f^*,p,w)/Q_{f*} \geq d$, the solution should be at the corner on this side and the global optimum is on the other side where $\Lambda(Q_f,p,w)/Q_f \geq d$. Then the investor is not small-pocket. This proposition considers the results conditional on the investor optimally chooses to have a small pocket.

(4) $\frac{dQ_j^*}{dW}\frac{ds_j^*}{dW} < 0$. *That is, safety and quantity always move to different directions when wealth changes.*

Proof. See Appendix B. The investor's care level can be higher than the socially optimum. When the investor is small-pocket, all the wealth is wiped out if damages occur (with probability $1-s$). The investor only cares about the cost and revenue when the firm stays (with probability s). For the same level of care s, the investor's effective marginal cost (the marginal cost "discounted" by the probability s) is smaller than the the social planner's marginal cost. We also know that the marginal revenue for the investor (the effective unit liability Λ/Q) is also smaller than the social planner's MR (unit damage d) by definition of "small-pockets." Therefore, for the socially optimum care level, MC(social)=MR(social), but MC(investor)<MR(investor) can happen, and in this case the firm's choice of care is greater than the socially optimum. □

The tort claimants bear some damages if the investor has a small pocket. When the firm increases care s, the tort claimants would bear more damages when tort is realized. This gives an extra incentive for the firm to invest more in care. Compared to the first best, the firm's choice of care can be higher. Below is a numeric example.

An example of overinvestment in care

Suppose $m(Q) = 7 - .2Q, C(s) = \frac{3}{1-s} - 12s, s \in [.5,1). d = 6, w = 20$, and $R_f = 1.01$. Social planner's problem is:

$$\begin{aligned}
&\max_{s,Q}\Big\{\int_0^Q [m(Q)-(1-s)d]dQ - R_f(1+C(s))Q\Big\}\\
=&\max_{s,Q}\Big\{\int_0^Q [7-.2Q-(1-s)*6]dQ - 1.01(1+\frac{3}{1-s}-12s)Q\Big\}\\
=&\max_{s,Q}\Big\{-.1Q^2-.01Q-\frac{3.03Q}{1-s}+18.12sQ\Big\}
\end{aligned}$$

First best solution $s^* = .59, Q^* = 16.45$.

Investor's problem:

$$\max_{s,Q}\Big\{(1-s)Q\Big(m(Q)+R_f[w/Q-(1+C(s))]-d\Big)^+ + sQ\Big(m(Q)+R_f[w/Q-(1+C(s))]\Big)\Big\}$$
$$=\max_{s,Q}\Big\{(1-s)Q\Big(7-.2Q+1.01[20/Q-(1+\frac{3}{1-s}-12s)]-6\Big)^+ + sQ\Big(7-.2Q+1.01[20/Q-(1+\frac{3}{1-s}-12s)]\Big)\Big\}$$

The solution is $s_f^*=.60, Q_f^*=14.17$ which satisfies $m(Q_f^*)+R_f[w/Q_f^*-(1+C(s_f^*))]=4.26<d(=6)$. That is, the investor has a small pocket. Notice that, with greater value of wealth, for example, when $w=40$ the choices become $s_f^*=.59, Q_f^*=8.23$. In this case, $m(Q_f^*)+R_f[w/Q_f^*-(1+C(s_f^*))]=9.01>d$ and the investor has a deep pocket.

Contrast to what people may believe that small pockets result in larger social inefficiency compared to deep-pocket investors - since they don't internalize damages - the equilibrium of a small-pocket can instead improve social efficiency and may even be close to the social optimum.

PROPOSITION 4. ***(No liability)*** *If $\Lambda(Q,p,w)=0$, compared to other liability cases, the firm invests least in care (zero) and most in scale, which can be greater or less than the first-best quantity depending on the demand elasticity.*

Proof. the first order conditions are

$$(s)\quad C'(s_0^*)R_f=0, \text{ and}$$
$$(Q)\quad m(Q_0^*)+m'(Q_0^*)Q_0^*=R_f(1+C(s_0^*))$$

$C(s_0^*)=0$ is immediate, thus $m(Q_0^*)+m'(Q_0^*)Q_0^*=R_f$.

To compared to other liability rules, here we take limited liability as an example. Under limited liability, the first order condition for scale Q is $m(Q_l^*)+m'(Q_l^*)Q_l^*=R_f\big(1+C(s_l^*)/s_l^*$. The inequality $R_f\big(1+C(s_l^*)\big)/s_l^*>R_f$ indicates $m(Q_l^*)+m'(Q_l^*)Q_l^*>m(Q_0^*)+m'(Q_0^*)Q_0^*$, and by Assumpt. C we have $Q_l^*<Q_0^*$. The proofs are similar for other liability cases.

Compared to first-best, the condition for scale greater than the first-best is

$$e_p(Q_0^*) > \left[1 - R_f/m(Q_s^*)\right]^{-1}.$$

□

Having no liability for damages seems unacceptable, but can happen. If the law system is not well established, powerful firms can evade liabilities easily. More commonly, tort may involve technologies that are not well understood or the diseases show their symptoms only chronically. The original investors may grab the revenue early and the original firms may shut down before the problem reveals. Tracing the parties who should be responsible after decades can be very costly, too. Even if the firm survives, should the tort claimants go after the current firms and investors? What if the firm has already advanced the technology and what if the management has changed? Going after the wrong group does not push incentives to the right direction and can damage the current business.

Considering such situation in which the firm is not responsible for any liability, tort is a pure externality and the investor has no incentives for care. It thus becomes a price-quantity tradeoff determined by demand elasticity as in the basic choice theory. It is more socially harmful when individual damage is large and demand is elastic, because compared to the social optimal choices, a more elastic demand would "internalize" more positive part while larger damage externalizes the negative part.[23] When the demand is not elastic, too large fraction of benefit goes to consumer surplus, and the firm would want to decrease the investment in scale. If the firm does not internalize enough damages, scale level can be too high instead.

3.1 The monopoly quantity: a special case with costless safety

It may be useful to compare the model to the intermediate microeconomics monopoly firm's problem, which is a special case when the firm is always safe. Then we have the following conclusion:

[23] With perfect price discrimination, the firm internalizing all the benefit would inevitably invest more compared to the "social optimum" because there is only negative externality.

PROPOSITION 5. ***(The monopoly quantity)*** *If $C(s) \equiv 0$, the firm invests in full safety and the same quantity compared to the no liability quantity, and higher quantity compared to the quantities under any other liabilities.*

Proof. Suppose Q_M^* is the equilibrium quantity when the firm is always safe, then the first order condition for quantity is $m(Q_M^*) + m'(Q_M^*)Q_M^* = R_f$. This is the same when there is no liability (i.e., $\Lambda(Q, p, w) = 0$) in which case the firm chooses minimal safety and the cost of safety is also zero. We can get the rest part of conclusion from Proposition 4. □

The choice of quantity is the highest without any consideration of safety, and is the same level when there is no liability at all. We can think of different liability rules as adding different caps to the liability. With lower caps the investor externalizes more benefits. The result is lower safety incentives and higher quantity incentives. Then there is some level of cap such that it is most socially beneficial along the line – the "second best" choice since we can never reach the first-best because of the market power. The choice of liability should be related to how close they are to this "second best."

The remaining part of this section provides a numerical example of the model and shows equilibria of different liability cases with different parameter values. I also add fixed cost to the model as a comparison. Some observations besides the above propositions are discussed.

3.2 An example of linear demand

In this example I assume linear demand. The cost function is assumed so that corner solutions are avoided.

$$
\begin{aligned}
m(Q) &= a - bQ \\
C(s) &= \frac{c}{1-s} + \frac{2\underline{s}c - (1+s)c}{(1-\underline{s})^2}, \quad s \in [\underline{s}, 1)
\end{aligned}
$$

For limited liability, we have the first order conditions

$$\text{when } \left(\frac{a-d}{b}\right)^+ \le Q < \frac{a}{b}$$

$$(s) \quad Q(s) = \frac{a}{b} - \frac{R_f}{b}C'(s) = \frac{a}{b} - \frac{cR_f}{b}\left[\frac{1}{(1-s)^2} - \frac{1}{(1-\underline{s})^2}\right]$$

$$(Q) \quad Q(s) = \frac{a}{2b} - \frac{R_f}{2bs}(1+C(s)) = \frac{a}{2b} - \frac{cR_f}{2bs}\left[\frac{1}{c} + \frac{1}{1-s} + \frac{2\underline{s}-(1+s)}{(1-\underline{s})^2}\right]$$

$$\text{when } 0 < Q \le \left(\frac{a-d}{b}\right)^-$$

$$(s) \quad s = 1 - \left[\frac{d}{cR_f} + \frac{1}{(1-\underline{s})^2}\right]^{-\frac{1}{2}}$$

$$(Q) \quad Q(s) = \frac{a}{2b} - \frac{R_f}{2b}(1+C(s)) - \frac{(1-s)d}{2b}$$

For unlimited liability, we have

$$\text{when } \left(\left(\frac{a-d}{2b} - \frac{R_f(1+C(s))}{2b}\right) + \sqrt{\left(\frac{a-d}{2b} - \frac{R_f(1+C(s))}{2b}\right)^2 + \frac{R_f W}{b}}\right)^+ \le Q < \frac{a}{b}$$

$$(s) \quad Q(s) = \left(\frac{a-sR_fC'(s)}{2b} - \frac{R_f(1+C(s))}{2b}\right) + \sqrt{\left(\frac{a-sR_fC'(s)}{2b} - \frac{R_f(1+C(s))}{2b}\right)^2 + \frac{R_f W}{b}}$$

$$(Q) \quad Q(s) = \frac{a}{2b} - \frac{R_f}{2b}(1+C(s))$$

$$\text{when } 0 < Q \le \left(\left(\frac{a-d}{2b} - \frac{R_f(1+C(s))}{2b}\right) + \sqrt{\left(\frac{a-d}{2b} - \frac{R_f(1+C(s))}{2b}\right)^2 + \frac{R_f W}{b}}\right)^-$$

$$(s) \quad s = 1 - \left[\frac{d}{cR_f} + \frac{1}{(1-\underline{s})^2}\right]^{-\frac{1}{2}}$$

$$(Q) \quad Q(s) = \frac{a}{2b} - \frac{R_f}{2b}(1+C(s)) - \frac{(1-s)d}{2b}$$

The following figures assume some parameter values. Besides the propositions discussed above, there are several observations:

Observation 1. *In Figure 3, when unit damage d is relatively small, full liability (endogenously chosen) has a worse outcome than limited liability because of inefficiency in scale. Limited liabil-*

ity results in less care but this inefficiency of care can be compensated by the improved efficiency in scale, making the choice more efficient overall. However, as the damages become bigger, society prefers higher care level and lower scale. In this case, limited liability generates too little incentives for care and too much incentives for scale, and full liability tends to be more efficient.

Observation 2. *Figure 4 shows an example that an investor with less wealth may end up with an equilibrium closer to the first best under unlimited liability rule. In the first graph, the investor with higher wealth level is deep-pocket and chooses an efficient care and inefficiently low scale under unlimited liability rule. In the second graph, however, the equilibrium is even more efficient than limited liability when the investor has less wealth and chooses to have a small pocket.*

Observation 3. *Figure 5 provides an observation of the sensitivity of firm's choices regarding changes of demand elasticity. With larger wealth* $W(=40)$*, demand elasticity has no impact on care but affects firm's choices of scale only proportionally. When demand is more elastic (which means a flatter* $m(Q)$*), society and investor both prefer higher level of scale and the choices of care and scale have the same sensibility to the change of elasticity. With limited wealth, however, the increment of scale as a result of higher elasticity of demand can be disproportionate with a sharper increase in scale under unlimited liability rule. This is suggesting that equilibrium under unlimited liability rule can be more sensitive subject to changes in demand, particularly when the investor has limited wealth.*

Observation 4. *Figure 6 compares different costs of care. Not surprisingly, increasing the prevention costs of damages lowers incentives for care for both the investor and society, and with unaffected scale level. This result may not be taken literally, since it does not necessarily hold true for every case. When the firm is small-pocket, the choices of the two dimensions can both change.*

3.3 With fixed cost

When the firm has fixed cost, average cost is decreasing. In this case the firm would not operate if they have to internalize all the damages, which would result in low quantity ex post. Ex ante, low

quantity makes average cost of investment higher than the marginal benefit, and the firm is less likely to start as a consequence. This can be good or bad depending on how big the damages are, shown by the two cases in Figure 7.

Observation 5. *Figure 7 shows that with full liability, investment in care reaches first-best but there is always underinvestment in scale (the first figure). The firm will even not start when there is fixed costs (the second figure). Limited liability under-provides care but increases scale and even makes possible some profitable projects to be undertaken.*

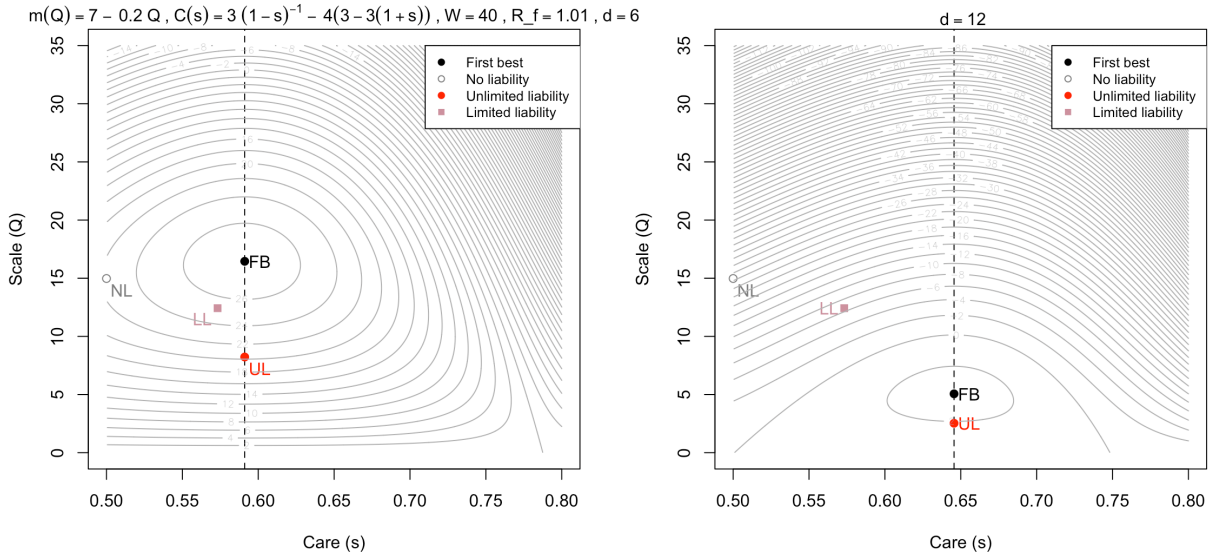


Figure 3: ***(Unit damage** d)* The x axis is care, and y axis is scale. The dots of different shapes and colors represent the equilibrium of different liability rules. The contour plot represents indifference curves of social value of the project. In these two graphs, the first graph shows the investor's choice when damage d is relatively small (=6), and the second graph shows a larger unit damage(=12). In these two examples, the investor is deep-pocket because wealth is big enough, and full liability is worse than limited liability in terms of social welfare because of inefficiency in under-investment in scale. However, as damages increase and other things equal, society prefers higher care level and lower scale. In this case, full liability is closer to then first best than limited liability.

So far I have analyzed a single firm who has market power and can set price for its product. The firm cannot fully internalize all the benefit from operation and the consumers obtain the "triangle" of the demand function. As a consequence, full liability can undermine the incentives to produce

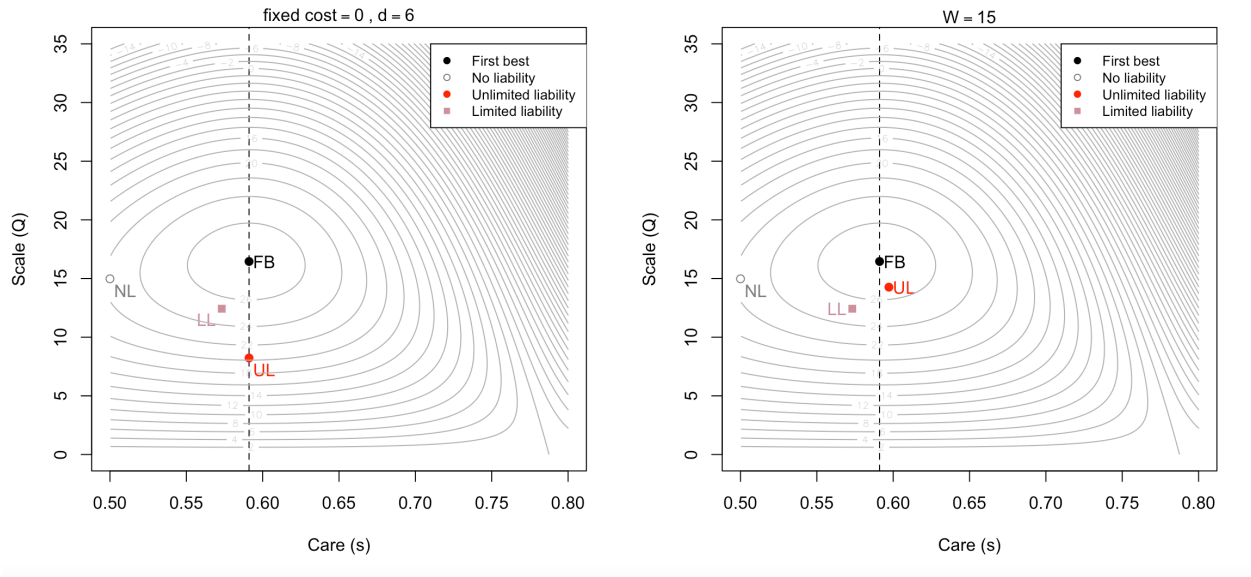
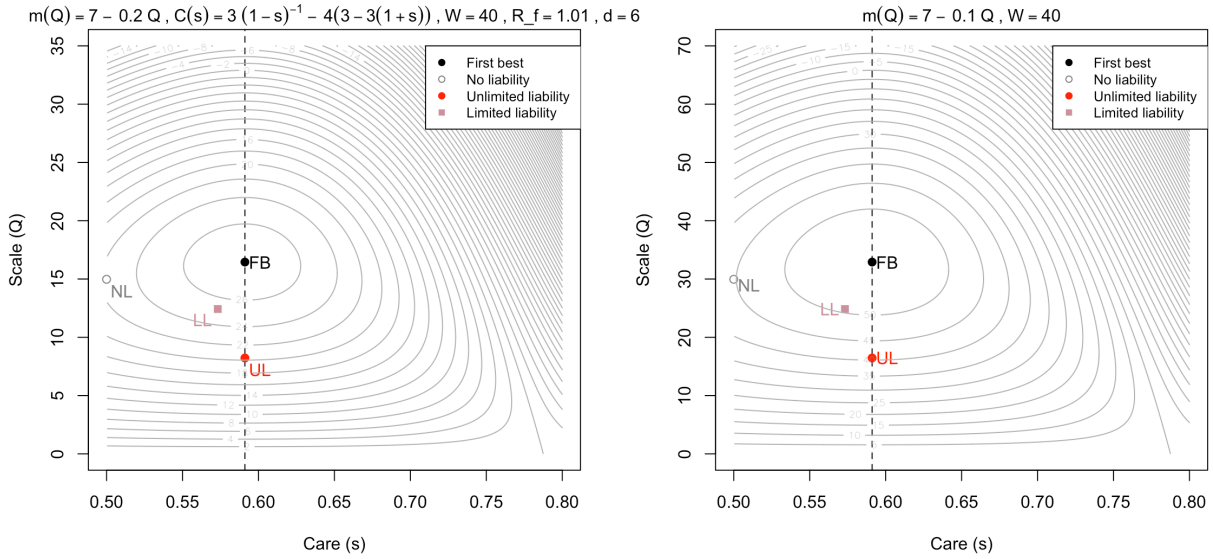


Figure 4: ***(Choice of deep-pocket or small-pocket)*** The notations follows the previous graphs. In these two graphs, the left graph shows the investor's choice when wealth W is relatively big (=40), and the right graphs shows a smaller wealth(=15). In the first graph, the investor with higher wealth level is deep-pocket and chooses an efficient care and inefficiently low scale under unlimited liability rule. In the second graph, however, the equilibrium is even closer to the first best when the investor has less wealth and chooses to have small pocket.

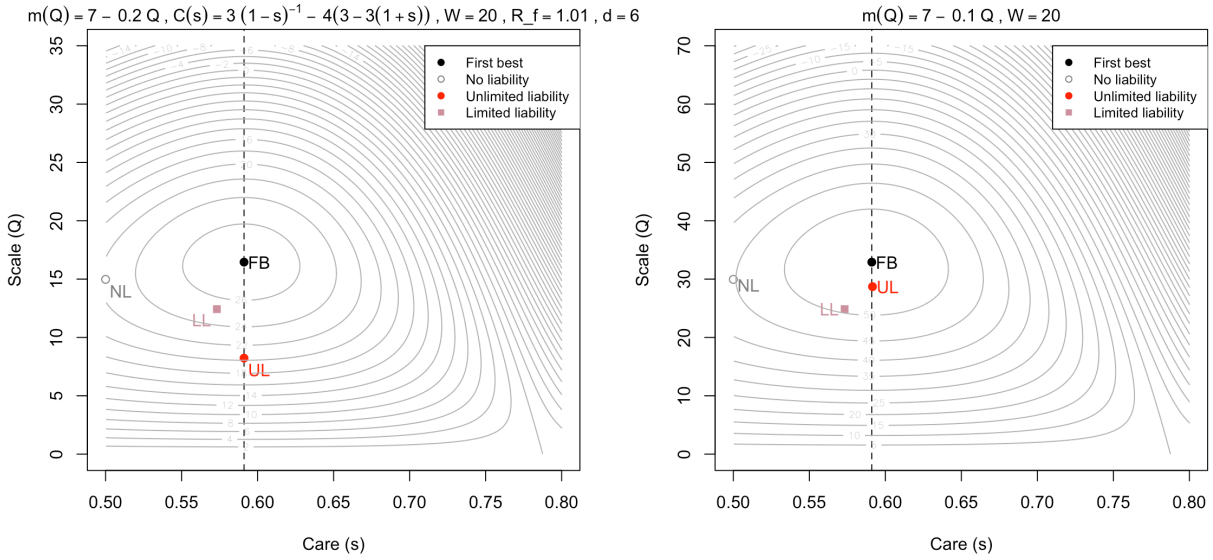
and can even halt a beneficial project especially when the demand elasticity is relatively low and the "triangle" is substantial. With fixed cost, the firm is likely not started at all. Limited liability can improve social welfare by increasing scale a large amount, and the social gain from it possibly offsets the social loss from increased probability of damage. The investor and the consumers benefit from limited liability, which hurts the potential tort claimants because their probability of suffering a loss is greater and they are not to be fully compensated. In the next section, I analyze how competition changes the equilibria and the implications of different liability rules.

4 Cournot Competition

I show in this section that with more competition, full liability tends to be more socially efficient than limited liability. For better discussion, I only focus on full liability and limited liability. It might be interesting to also talk about the investor with small pockets, but the comparisons between



(a) ***Change demand elasticity with larger wealth.*** *With larger wealth $W(=40)$, demand elasticity has no impact on care but affects firm's choices of scale only proportionally. When demand is more elastic (flatter $m(Q)$), society and investor both prefer higher level of scale.*



(b) ***Change demand elasticity with smaller wealth.*** *When demand become more elastic, scale increases for every equilibrium, but can be disproportionate with a sharper increase of scale under unlimited liability rule. This is suggesting that equilibrium under unlimited liability can be more sensitive to changes in demand, particularly when the investor has limited wealth.*

Figure 5: ***(Change demand elasticity with high/low level of wealth)***

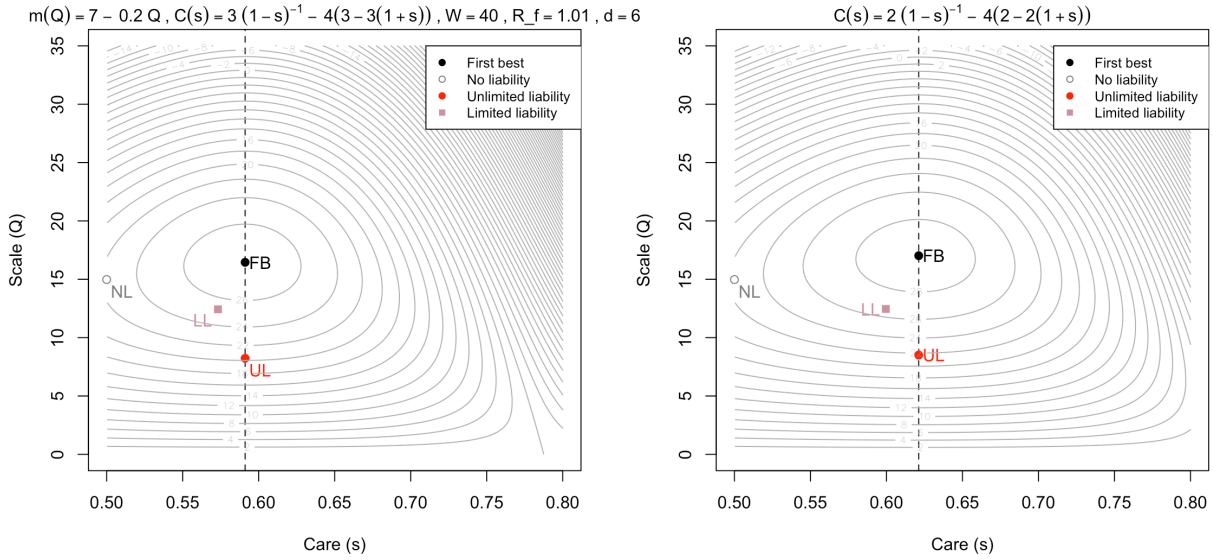


Figure 6: ***(Cost of care)*** The notations follows the previous graphs. When it is more expensive to prevent damages, incentives for care decrease for everyone and incentives for scale do not change.

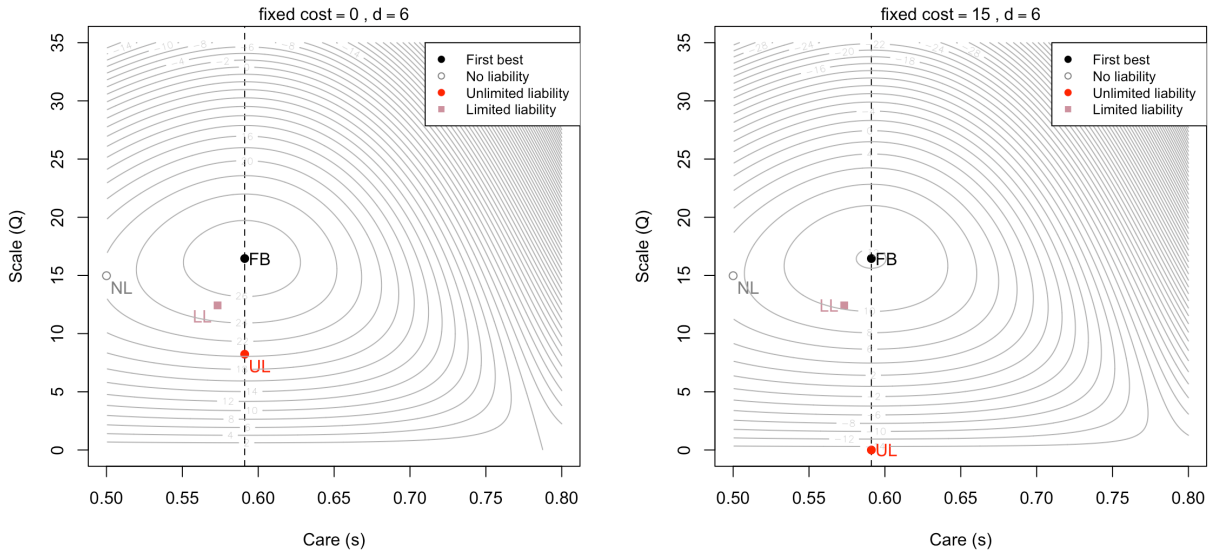


Figure 7: ***(Change fixed cost)*** The notations follows the previous graphs. With full liability, investment in safety reaches first-best but there is always underinvestment in quantity (figure on the left). The firm will even not start when there is fixed costs (figure on the right). Limited liability under-provides safety but increases quantity compared to full liability.

limited full liability is more intuitive. The formal investor's problems also include the possibility of small pockets for the readers who are interested in this case. The results can also be applied to the labor market and is discussed in more detail in a later section. If the market is very competitive instead, full liability is close to the first best because scale is also efficient. Competition intensity is measured by the number of firms. Suppose there are N homogeneous firms investing in the same technology, and a representative firm i chooses care s_i and scale Q_i. In the simplest case the firm's choice of safety is independent of other firms' choices.[24] The social welfare function is the same as (3), and firm i's problem is

$$\max_{s_i,Q_i}\left(Q_i p-(1-s_i)\Lambda(Q_i,p,w_i)+w_i\right) \tag{13}$$

$$\text{s.t.}\quad p=m(Q_{-i}+Q_i),0\leq Q_i \tag{14}$$

$$w_i\equiv R_f[\frac{W}{N}-Q_i(1+C(s_i))]>0 \tag{15}$$

$$\text{and}\quad \underline{s}\leq s_i<1 \tag{16}$$

Where

$$\Lambda(Q_i,p,w_i)=\begin{cases}0, & \text{if no liability}\\ (Q_i p)\wedge(Q_i d), & \text{if limited liability}\\ \left(Q_i p+w_i\right)\wedge(Q_i d), & \text{unlimited liability with small pockets}\\ Q_i d, & \text{unlimited liability with deep pockets}\end{cases}$$

In equilibrium, all the firms choose the same safety and productivity levels, i.e., $s_i=q,Q_i=I/N$. See Table 3 for detailed computation of the first order conditions. We then have the following proposition:

PROPOSITION 6. ***(Cournot competition)*** *When the number of firms increases to infinity, equi-*

[24] In some cases, though, a tort litigation would trigger a series of litigation on similar products which are produced by other firms.

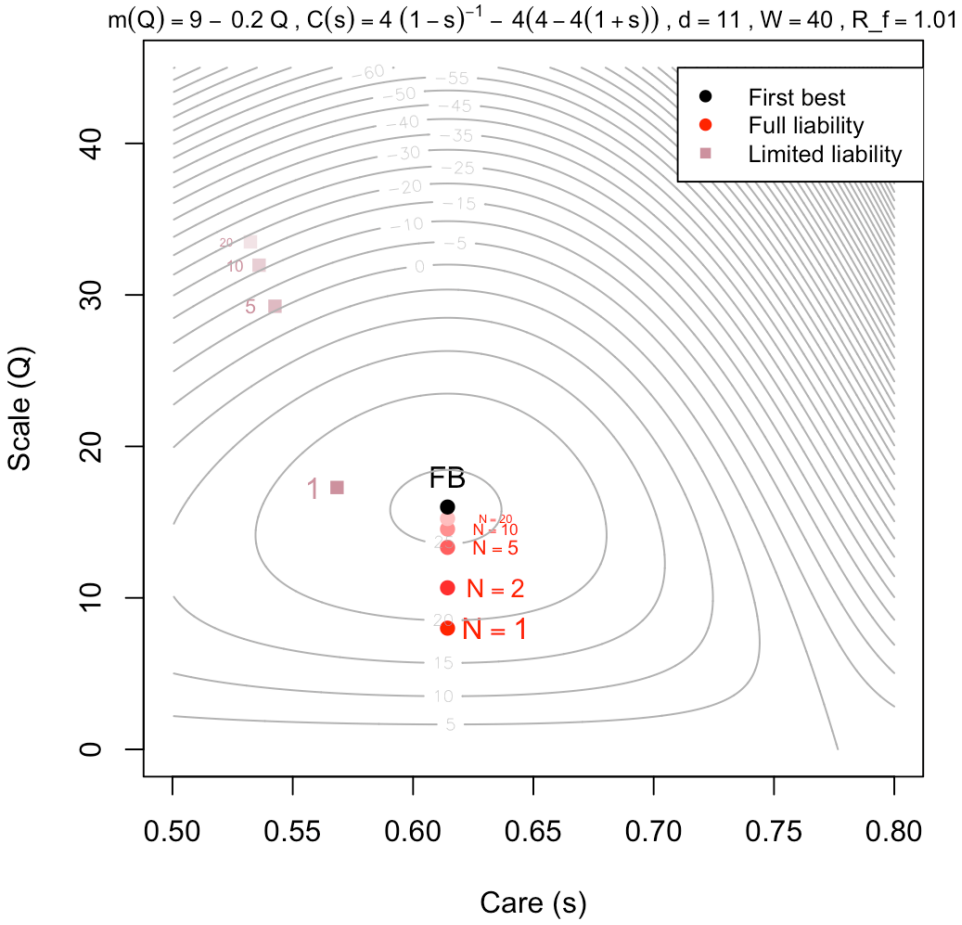


Figure 8: ***(Cournot competition)*** The notations follow the previous figures. When the number of firms increases (N =1,2,5,10,20 shown in the graph with the same shapes and colors but reduced saturation), equilibrium of full liability converges to first best (red dots), whereas with limited liability the equilibrium deviates from first best and result in over-provision of scale and under-provision of care.

librium of full liability converges to first-best, whereas limited liability diverges from the first best and results in overinvestment in scale and underinvestment in care and is socially inefficient.

Proof. The first order conditions for full liability are

$$(s_i) \quad s_i^* = s_s^*$$

$$(Q_i) \quad m(Q_u^*) + m'(Q_u^*)Q_u^*/N - (1 - s_i^*)d - R_f(1 + C(s_i^*)) = 0.$$

We can compute how Q_u^* changes when the number of firms increases:

$$\frac{dQ}{dN} = \frac{Q/N}{N+1+\frac{m''(Q)Q}{m'(Q)}}.$$

Equation (12) indicates that $\frac{dQ}{dN} > 0$, so the equilibrium quantity is increasing when the number of firms increases. As $N \to \infty$,

$$m(Q_u^*) + m'(Q_u^*)Q_u^*/N - (1-s_i^*)d - R_f(1+C(s_i^*)) \to m(Q_u^*) - (1-s_i^*)d - R_f(1+C(s_i^*))$$

since Q cannot be infinite. When the number of firm increases, while safety is always optimal, the first order condition for quantity is closer to first-best. We can do the same calculation for other liability rules. See Appendix C. □

At extreme, the market becomes perfect competitive. Demand elasticity is perfect. The efficiency of full liability comes immediately from the fact that the firm internalizing all the benefits and costs fully aligns its interest with that of society. However, limited liability which does not fully internalize damages is more inefficient with increased competition and would result in inefficiency of under-investment in care and overinvestment in scale. In order for full liability under intense competition to work, one also has to make sure that the investors have enough assets to cover all the liability, that is, the investor has to be deep pocket, otherwise the evasion of liability would make investor's choices less socially efficient. The equilibrium would deviate from the first best with more competition and is similar to the case under limited liability.

5 Lease versus Buy

In the previous section, the only assets in the firm available to pay tort claimants are the proceeds from selling the product. This is true if the firm rents capital or uses debt to finance its capital and can pledge the capital to ensure that the repayment to the lender has higher priority in resolution.

(1)	(2)	(3)	(4)
rule	$\Lambda(Q,p,w)$	FOC:(s)	FOC:(Q)
FB	-	$C'(s_s^*)=\frac{d}{R_f}$	$m(Q_s^*)\leq R_f\left(1+C(s_s^*)\right)$ $+(1-s_s^*)d$
FL/UL	Qd	$C'(s_u^*)=\frac{d}{R_f}$	$m(Q_u^*)+m'(Q_u^*)Q_u^*/N\leq$ $R_f\left(1+C(s_u^*)+k\right)+(1-s_u^*)d$
LL	$Qp+Qk(<Qd)$	$C'(s_l^*)=\frac{m(Q_l^*)+k}{R_f}$	$m(Q_l^*)+m'(Q_l^*)Q_l^*/N+k\leq$ $\frac{R_f\left(1+C(s_l^*)+k\right)}{s_l^*}$
UL-SP	$Qp+R_f(W/N-Q(1+C(s))-Qk)(<Qd)$	$C'(s_j^*)=$ $\frac{m(Q_j^*)+R_f(W/Q-1-C(s_j^*)-k)+k}{s_j^*R_f}$	$m(Q_j^*)+m'(Q_j^*)Q_j^*/N\leq$ $R_f\left(1+C(s_j^*)+k\right)$

red: adding number of firms.
blue: adding capital installation.

Table 3: ***(Comparisons of variables: increase competition and add capital)*** When the number of firms N increases to infinity, equilibrium of full liability converges to first-best, whereas limiting liability (either because of the limited liability rule or the investors having small pockets) results in too high investment in scale and too low investment in care and is socially inefficient.
Requiring purchased capital as an example of general capital requirement may push the equilibria back to first-best for limited liability, but either do not change incentives for small pockets under unlimited liability or tend to be inefficient when the investor has deep-pocket.

In tort litigation, the firm has to return the rental capital or transfer the capital to the lender so that it is out of reach of the tort claimants. This section discusses the situation in which the firm can only buy capital to produce. This may happen when the operating capital is very specialized (especially for new technology) and the firm may not be able to rent existing capital or find lenders to provide funds. It may also come from capital requirement for regulatory purpose or requirement by other contractual creditors as a cushion. In this section I assume proportional capital requirement, namely, each unit of production requires k units of capital. Capital does not depreciate when tort occurs as a simplification in the model, but if it does (probably because capital is firm specific and should be liquidated with loss), the loss would make rental price higher if priced out. With firm purchasing capital, this depreciation is fully absorbed by the tort claimants in the limited liability case, and can be partially or fully borne by the investor under unlimited liability. I focus on the case in which there is no depreciation and the firm can sell the capital at the original value kQ_i.

The social welfare function is the same as (3). Firm i's problem is

$$\begin{aligned}
&\max_{s_i,Q_i}\left(Q_i p-(1-s_i)\Lambda(Q_i,p,w_i)+w_i\right)\\
&\quad \text{s.t.} \quad p=m(Q_{-i}+Q_i), 0\leq Q_i\\
&\qquad\quad w_i\equiv R_f[W/N-(1+k+C(s_i))Q_i]>0\\
&\quad \text{and} \quad \underline{s}\leq s_i<1
\end{aligned}$$

Where

$$\Lambda(Q_i,p,w_i)=\begin{cases}
0, & \text{if no liability}\\
[Q_i(p+k)]\wedge(Q_i d), & \text{if limited liability}\\
\left(Q_i p+Q_i k+w_i\right)\wedge(Q_i d), & \text{unlimited liability with small pockets}\\
Q_i d, & \text{unlimited liability with deep pockets}
\end{cases}$$

PROPOSITION 7. ***(Buying capital)*** *If capital is purchased, more damages are internalized under*

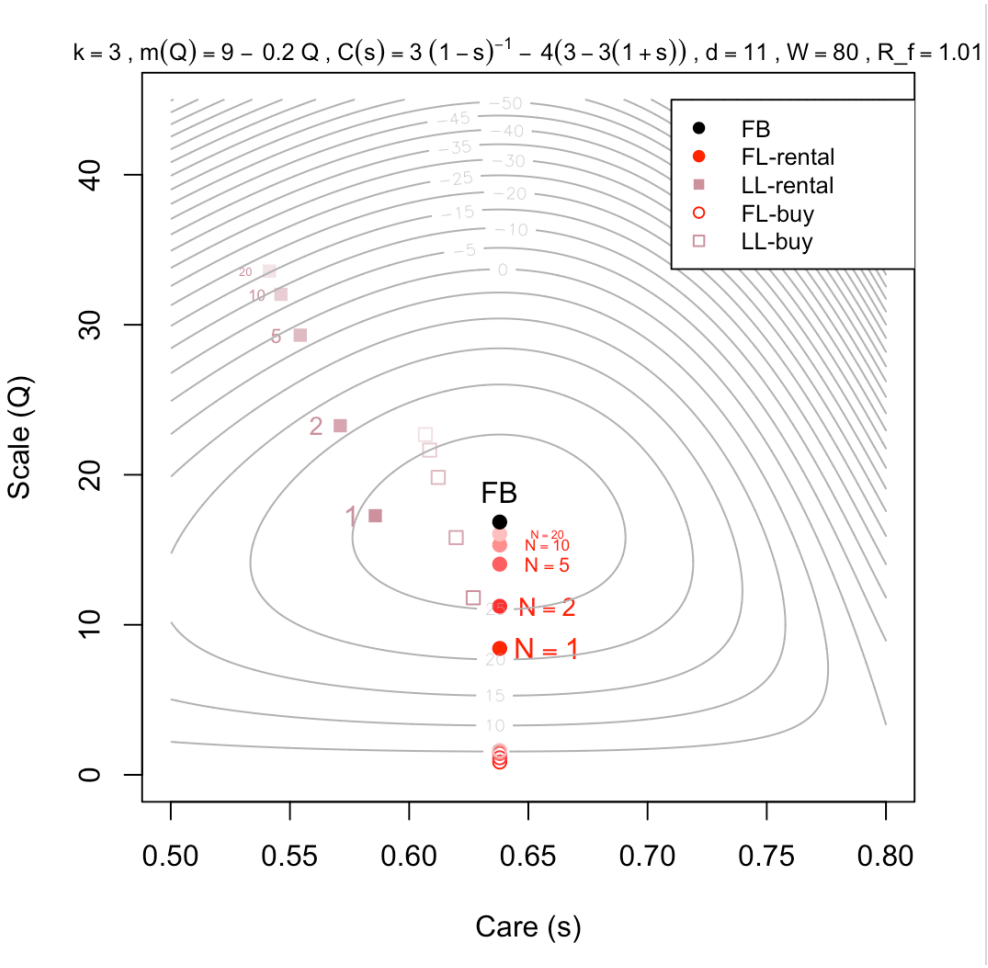


Figure 9: ***(Lease versus buy)*** The solid shapes represent the same as in Figure 8. The hollow shapes represent the relevant liability rules with purchased capital instead of rental capital. The plot shows that requiring purchased capital may push the equilibrium to first-best for limited liability shown by the hollow pink dots, but tends to be inefficient when the investors are deep-pocket (depicted by the red hollow dots).

limited liability so that equilibrium is pushed towards first-best with sufficient competition. Capital installation does not change the equilibria of unlimited liability with small pockets, but pushes the equilibria of deep pockets away from the first-best and may even turn down the investment.

Proof. Limited Liability requires $m(Q_l^*) < d - k$ and the first order conditions

$$\begin{aligned}
&(s_i) \quad m(Q_l^*) = -k + R_f C'(s_l^*) \\
&(Q_i) \quad m(Q_l^*) + m'(Q_l^*)Q_l^*/N = -k + R_f(1 + k + C(s_l^*))/s_l^*.
\end{aligned}$$

Detailed computation see Table 3. It is easy to prove that $\frac{dQ_l^*}{dk} > 0$ for the first equation and $\frac{dQ_l^*}{dk} < 0$ for the second, hence the function $Q_l^*(s_l^*)$ shifts upward for the first equation, and shifts downward for the second as k increases, resulting in lower s_l^* and higher Q_l^*. With full liability, we have

$$\begin{aligned}
&(s_i) \quad s_i^* = s_s^* \\
&(Q_i) \quad m(Q_u^*) + m'(Q_u^*)Q_u^*/N - (1 - s_i^*)d - R_f(1 + k + C(s_i^*)).
\end{aligned}$$

Since $\frac{dQ_l^*}{dk} < 0$ for the second equation, the equilibrium care level does not change but the scale drops as k increases. For unlimited liability:

$$\begin{aligned}
&\text{if } m(Q_u^*) + R_f(W/Q_u^* - 1 - k - C(s_u^*)) < d - k \\
&\qquad (s_i) \quad m(Q_u^*) + k + R_f(W/Q_u^* - 1 - k - C(s_u^*)) - s_u^* R_f C'(s_u^*) \\
&\qquad (Q_i) \quad m(Q_u^*) + m'(Q_u^*)Q_u^*/N + k - R_f(1 + k + C(s_u^*))
\end{aligned}$$

When R_f is close to 1, the problem is close to the problem with rental capital. It is true that capital in the firm does not earn risk-free interest outside of the firm, but if the difference is negligible, capital requirement would only act as a fixed cost and shut down the firm if sufficient. □

With unlimited liability, requiring (purchased) capital in effect adds overhead, either a fixed number or in this setting a proportional cost, to the investor. As shown in Figure 9, the deep

pockets still invest in first-best care but underinvestment in scale. It would not change the equilibrium for small pockets because the capital inside or outside of the firm can always be seized under unlimited liability rule. However, requiring capital internalizes damages for limited liability, increasing incentives for care and at the same time decreasing incentives for scale and improves social welfare particularly when there is more competition.

Notice that, minimum capital requirement also moves the budget down, and if sufficient enough, makes some equilibria above the budget line infeasible. In this case, investments in care and scale for limited liability may move along the budget line. Along the line, there is also a substitution effect of care and scale.

6 Other Stakeholders

Previously, consumers are the only stakeholders of the firm. The inefficiency of underinvestment in quantity is a result of firm's market power in the product market that externalizes benefits, and as discussed before, limited liability has a flavor of Ramsey pricing and may enhance social welfare by "subsidizing" the firm through reducing the liability from damages. This is also true if the firm has other stakeholders. For example, large corporations usually have big impact on communities where the firms located in. They create jobs, provide investment opportunities, safety, unique community identity, economic health and development, etc. Much of the characteristics are valuable but not captured by the profits. Another aspect is the discrepancy of management interest and shareholder interest, which are not necessarily align because of separation of ownership and control. Management nowadays usually do not take full liability or even exempt from liabilities on the consequences of decision making, except for fraudulent conveyance and breach of duty. Part of the reason is that they do not capture all the benefits in the firm and therefore bearing full liability would probably result in too conservative investment strategy and may let go of profitable investment opportunities. In this section, I discuss two other firm's stakeholders: governments and employees.

6.1 Government as a stakeholder: taxation

Governments are considered as a major stakeholder of a corporation because they collect corporate income taxes from the firm, payroll taxes from the employees, as well as other taxes (sales taxes, etc.). In some states, certain corporations also pay franchise taxes for the right to be chartered. With higher taxes, the firm externalizes larger proportion of benefits. A lump-sum tax such as a franchise tax would be similar to a fixed cost to a firm. It may not distort incentives once the firm is established, but it makes it less attractive to start the firm in the first place. I assume unit tax with tax rate $\tau \in [0,1)$. The investor's problem is

$$\max_{s_i,Q_i} \left(\left[Q_i p - (1-s_i)\Lambda(Q_i,p,w_i) \right] + w_i - \text{Tax} \right) \tag{17}$$

$$\text{s.t.} \quad \text{Tax} = \left[(1-s_i)(Q_i p - \Lambda(Q_i,p,w_i))^+ + s_i Q_i p \right] \tau \tag{18}$$

$$p = m(Q_{-i} + Q_i), \quad 0 \le Q_i \tag{19}$$

$$w_i \equiv R_f[W/N - (1+C(s_i))Q_i] > 0, \tag{20}$$

$$\text{and} \quad \underline{s} \le s_i < 1 \tag{21}$$

Where

$$\Lambda(Q,p,w_i) = \begin{cases} 0, & \text{if no liability} \\ Q_i p \wedge Q_i d & \text{if limited liability} \\ \left(Q_i p + w_i \right) \wedge Q_i d & \text{unlimited liability (small pocket)} \\ Q_i d, & \text{unlimited liability (deep pocket)} \end{cases}$$

Figure 10 shows an example when per unit taxes are 0, 0.25 and 0.5, respectively.

PROPOSITION 8. ***(Taxation)*** *Full liability results in underinvestment in quantity when there is taxation. With limited liability, increasing unit taxes undermines both quantity and quality incentives.*

Proof. See Appendix D. □

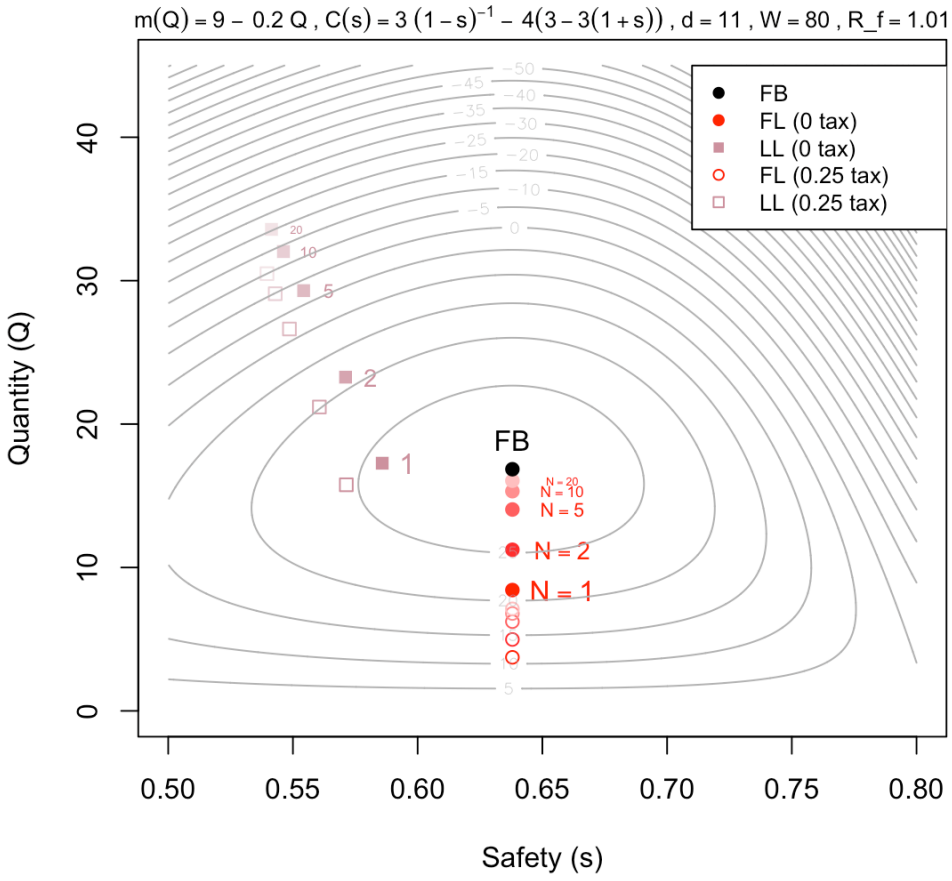


Figure 10: ***(Taxation)*** The solid shapes represent the same as in Figure 8 in which there is no tax. The hollow shapes represent the relevant liability rules with unit tax .25. Full liability results in underinvestment in quantity when there is taxation. When liability is capped, increasing unit taxes undermines both quantity and quality incentives.

Higher tax rate would externalize more benefits and results in lower quantity for all cases. When liability is capped, taxation results in less assets in the firm to compensate tort claimants. This also discourages safety incentives. As shown in Figure 10, taxation is bad socially with deep pockets because the firm may not start. Yet it is not necessarily bad socially when liability is capped, especially when there is more competition. With intense competition, quantity can be way too high.

6.2 Employees as stakeholders

In the model I focus on the product market, but we can have similar analysis on the labor market. If the firm has monopsony power in the labor market, then the firm does not capture the full benefits of people working in the firm and will tend to operate in a smaller scale. Monopsony is not unusual in the U.S. labor market. A typical example is a mining town in the mountains, where it is remote and has only few mining employers. If the firm is the only employee, the marginal cost is bigger than the workers' reservation utility, because to hire one more worker the wage has to increase for every worker that the firm hires. This probably partly explains small scales of firms in small places where firms have monopsony/monopoly power in the labor market as well as other factor markets. Similar to the conclusion before, if there is also fixed cost, the firm probably would not start in the first place. Beneficiaries have long advocated for unionization and increased wages to a level comparable to a competitive outcome to achieve a more "equitable economy," and the thought can be traced back to as early as Robinson (1933), but this would externalize more benefits and consequently reduces demand for labor and social welfare.

Our model suggests an analysis parallel to the product market analysis: limited liability would mitigate the inefficiency of not internalizing all the benefits, and social efficiency in quantity may be improved. Increasing competition only improves social welfare when the firm bears full liability, which requires unlimited liability and investors have deep pockets. Increasing competition will result in too low safety and too high quantity when liability is capped. With proper capital requirement, the equilibrium under limited liability may be drawn towards the first-best, but that does not work for unlimited liability.

7 Conclusion

This paper provides a theoretical framework to study choice of care (safety) and scale (quantity) under limited and unlimited liability rules. When the firm's other stakeholders obtain large benefits from the firm, full liability results in under provision of quantity. I mainly focus on the product

market in which such inefficiency is a result of big consumer surplus when the firm has monopoly power and faces a less elastic demand. We can also extend the discussion when the firm has other stakeholders, such as communities, governments, and when the firm has market power in other markets such as the labor market. Limited liability mitigates the inefficiency caused by externalization of benefits, because it reduces the damages taken by the investors as a means of "subsidy." An actual subsidy to a firm may not be possible in the real world for social and political reasons. Alleviating liability for damages can be an easier way to increase incentives for scale.

With intensified competition, firms capture higher fraction of benefits would have equilibrium converge to the first best under full liability, but that also requires the investors to be deep-pocket. If the investors have limited liability, the equilibrium would overinvest in scale and underinvest in care, which is also true when the investor has a small pocket under unlimited liability. However, one advantage of limited liability is that it is flexible to include other policies to adjust for cross firm differences. For example, minimum capital requirements, requiring insurance, and setting up funds as a buffer increases liability paid by the firm.

So far I have only studied the single investor's problem. Even when competition is discussed, each firm only has one investor because the intention is to focus on the effect of reduced market power. In practice, there is still more question to ask: how to allocate liability for firms with multiple shareholders if an extra liability is ideal? One possible solution could be to adopt the "joint and several" unlimited liability rule, which is commonly used in partnerships, where the creditors can go after the deep-pocket investors. This rule works better for closely held corporations but may not be practical for public corporations as it requires significant information gathering and verification. Additionally, the deep pockets could hide their assets strategically (such as through as trust fund) or find someone without assets to delegate their investments. Alternatively, a "pro rata" liability rule, which assigns liability proportional to shareholding, could largely solve the information problem. This could be implemented by requiring partial liability coverage on top of the limited liability rule. However, the question of how to allocate liability becomes more complex when considering shareholders whose interests do not align, as well as bondholders who may also

be responsible. These issues are not addressed in the current paper and is left for future studies.

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A

The firm's problem is to maximize

$$(1-s)Q\Big(m(Q)-d\Big)^{+}+sQm(Q)+R_fQ[W/Q-\big(1+C(s)\big)]$$

The first order conditions are

$$\begin{aligned}
&\text{if } m(Q)^{+}\leq d\\
&\qquad (s)\quad m(Q)=R_fC'(s)\\
&\qquad (Q)\quad m(Q)+m'(Q)Q=R_f(1+C(s))/s\\
&\text{if } m(Q)^{-}\geq d\\
&\qquad (s)\quad d=R_fC'(s)\\
&\qquad (Q)\quad m(Q)+m'(Q)Q-R_f(1+C(s))-(1-s)d=0
\end{aligned}$$

When $m(Q)<d$,

$$m(Q_l^*)=\frac{R_f(1+C(s_l^*))/s_l^*}{1-1/e_p(Q_l^*)} \tag{22}$$

is derived from the first order condition above.

When $e_p(Q_l^*)>\big[1-\frac{R_f\left(1+C(s_l^*)\right)}{s_l^*m(Q_s^*)}\big]^{-1}$, we have $Q_l^*<Q^*$.

B

The firm's problem is to maximize

$$(1-s)Q\Big(m(Q)+R_f[W/Q-\big(1+C(s)\big)]-d\Big)^{+}+sQ\Big(m(Q)+R_f[W/Q-\big(1+C(s)\big)]\Big)$$

The first order conditions are

$$
\begin{aligned}
&\text{if } m(Q)+R_f(W/Q-1-C(s))<d \\
&\qquad (s) \quad m(Q)+R_f(W/Q-1-C(s))-sR_fC'(s)=0 \\
&\qquad (Q) \quad m(Q)+m'(Q)Q-R_f(1+C(s))=0 \\
&\text{if } m(Q)+R_f(W/Q-1-C(s))\geq d \\
&\qquad (s) \quad d-R_fC'(s)=0 \\
&\qquad (Q) \quad m(Q)+m'(Q)Q-R_f(1+C(s))-(1-s)d=0
\end{aligned}
$$

Since $m(Q_j^*)+R_f(W/Q_j^*-1-C(s_j^*))<d, C'(s_j^*)s_j^*<C'(s_s^*)$ indicating that $s_j^* \lesseqgtr s_s^*$. Then $m(Q_j^*)+m'(Q_j^*)Q_j^*<m(Q_s^*)$ and Q_j^* can be either greater or less than Q_s^*.

For the second statement, if $Q_j^*<Q_l^*$, then $m(Q_j^*)+R_f(W/Q_j^*-1-C(s_j^*))>m(Q_l^*)$, indicating that $C'(s_j^*)>C'(s_l^*)$, thus $s_j^*>s_l^*$.

For (3), d does not enter the first order conditions when $m(Q_j^*)+R_f(W/Q_j^*-1-C(s_j^*))<d$. It affects the threshold of being judgment-proof. However, It is uncertain how W affects the threshold because s and Q would also change.

Specifically, when $W\uparrow$, $s\downarrow$ indicates that $Q\uparrow$, otherwise the FOC for s does not hold equal. This can also be confirmed by the FOC for Q following assumption C. If $s\uparrow$ instead, the FOC for Q suggests that $Q\downarrow$.

C

We can do the same calculation for limited liability: when $m(Q_l^*) < d$,

$$
\begin{aligned}
(s_i) \quad & m(Q_l^*) - R_f C'(s_l^*) \\
(Q_i) \quad & m(Q_l^*) + m'(Q_l^*)Q_l^*/N - R_f(1+C(s_l^*))/s_l^*. \\
\frac{dQ}{dN} &= \frac{I/N}{N+1+\frac{m''(Q)Q}{m'(Q)}+\frac{m'(Q)Q}{sR_fC''(s)}} \\
\frac{ds}{dN} &= \frac{m'(Q)}{R_fC''(s)}\frac{dQ}{dN}
\end{aligned}
$$

When N is sufficiently large, $\frac{dQ}{dN} > 0$ and $\frac{ds}{dN} < 0$. This is similar for unlimited liability with judgment-proof investors.

D

With limited liability, firm's problem is to maximize

$$(1-\tau)(Q_i p - (1-s_i)\Lambda(Q_i, p, w_i)) + R_f[W/N - (1+C(s_i))Q_i]$$

The first order conditions are

$$
\begin{aligned}
&\text{if } m(Q) < d \\
&\qquad (s_i) \quad (1-\tau)m(Q)Q - R_f C'(s)Q = 0 \\
&\qquad (Q_i) \quad (1-\tau)s(m(Q) + m'(Q)Q/N) - R_f(1+C(s)) = 0 \\
&\text{if } m(Q) + R_f(W/Q - 1 - C(s)) \geq d \\
&\qquad (s) \quad d - R_f C'(s) = 0 \\
&\qquad (Q) \quad m(Q) + m'(Q)Q - R_f(1+C(s)) - (1-s)d = 0
\end{aligned}
$$