Limited liability and investment*

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ABSTRACT

Why do firms causing damages enjoy limited liability? This paper builds a theoretical framework to study a firm's preference for safety and scale under different liability rules. When the firm has positive externalities received by other stakeholders (investors, consumers, employees, suppliers, communities, government, etc.), full liability is socially inefficient because it undermines the incentives to invest in quantity and even halts beneficial projects. Limiting liability externalizes some damages, and may enhance social welfare by mitigating the externalization of benefits, similar to Ramsey pricing. As positive externalities fall, the equilibrium of full liability converges to the first-best, while the equilibria of limited liability (either constrained by the limited liability rule or by investors having small pockets) deviate from the first-best and encourage underinvestment in safety and overinvestment in quantity. Therefore, limited liability is not a one-size-fits-all policy that achieves the optimum for different firms. This opens up possibilities of other rules that would improve incentives.

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1 Introduction

When problems of asbestos began to be well known by the public after decades since the problems emerged, the Manville Corporation (formerly Johns-Manville) was subject to over 16,500 tort[1] lawsuits pending against it by the year 1982, a result of "an average filing rate of 3 cases per hour, every hour of every business day."[2] Manville filed for Chapter 11 bankruptcy in that year and set a pool of funds in excess of $2 billion using its future profits and contributions from insurers, with current payout percentage of only 5.1% as of today.[3] As in any other mass tort litigation of public corporations, none of Manville's shareholders were held personally liable for the damage caused by the firm.[4] Though "piercing the corporate veil" provides an exception of limited liability, it is more related to closely held corporations and the veil piercing cases are generally rare, messy and unpredictable.[5] Besides corporations, limited liability also applies to other business entities including limited liability companies (LLCs) and even partnership-like firms (LPs, LLPs and LLLPs).[6] The question is not new: should the investors in these enterprises that potentially cause damages also have the "privilege" of limited liability?

[1]Definition of "tort" from Merriam-Webster.com Dictionary: "a wrongful act other than a breach of contract for which relief may be obtained in the form of damages or an injunction. Its root meaning of 'twisted' (as opposed to 'straight') obviously came to mean 'wrong' (as opposed to 'right'). Torts include all the so-called 'product-liability' cases, against manufacturers of cars, household products, children's toys, and so on. They also cover dog bites, slander and libel, and a huge variety of other very personal cases of injury, both mental and physical...If you're sued for a tort and lose, you usually have to pay 'damages' – that is, a sum of money – to the person who you wronged." Typically, the tort claimants cannot contract with the firm before damages occur.

[2]Delaney (1992) has a chapter of detailed discussion of the case and the firm's bankruptcy resolution. Asbestos is a fibrous mineral material proved to be great for home insulation and fireproofing, but if a single fiber is inhaled, it will be embedded in the lung and over time may lead to several types of lung cancer. These problems began to emerge as early as 1906, but the asbestos manufacturer only claimed to know the damage not until 1964.

[3]`https://www.mesothelioma.com/asbestos-exposure/companies/johns-manville/`

[4]e.g. A.H. Robins and the Dalkon Shield; Union Carbide and Bhopal; Johns-Manville and asbestos; Exxon and the Valdez oil spill; Dow-Corning and silicone breast implants, Roe (1986) provides rationales and evidence that public firms subject to mass tort all file for Chapter 11 and with even the valuable assets in the firm untouched. A more recent work Ramberg (2011) notes empirical evidence that no public corporation has ever been pierced the corporate veil.

[5]A famous exception of veil piercing case in the U.S. is *Walkovszky v. Carlton* (Walkovszky v. Carlton - 18 N.Y.2d 414, 276 N.Y.S.2d 585, 223 N.E.2d 6 (1966)). Carlton, shareholder of ten cab corporations each owned only two taxis. When a taxi-cab struck the pedestrian Walkovszky, the corporation that the cab belongs to had only minimum liability insurance required by law ($10,000), not enough to cover the tort liability. In this case, Carlton was not held personal liable for the accident, and thus his assets in other nine corporations remained inaccessible.

[6]Limited liability partnerships (LLPs) and limited liability limited partnerships (LLLPs) are naturally having limited liability, as a result of expansion of limited liability in the recent decades. Limited partnerships (LPs) usually set corporations as general partners as a shield against liability.

To answer the question, I provide a theoretical framework to study investors' choices of safety and scale (or quantity) under limited and unlimited liability rules. Limited liability is at firm level, but we can effectively get the same impact with unlimited liability if the investor is judgment proof.[7] The focus of this paper is on a pure tort case in which all the negative externalities are in this tort. It is hard to contract on the liability ex ante, therefore liability rules can be used as a means of regulation. The results show that if firm's other stakeholders obtain big surplus from operation – consumers who are paying less than their reservation prices, workers who are paid more than the reservation wage, beneficiaries from spending from higher taxes, or communities in which there are less crime – the investor bearing full liability would invest less in quantity compared to the first-best, and the firm may not even get started.[8] Contrary to a widely made argument that limited liability should be prevented by society as a shield to externalize risk and costs,[9] I find that the limitation of damages may enhance social welfare by mitigating the inefficiency of under provision if there are positive externalities to other stakeholders. However, when the other benefits fall (for example, with more competition in the product market or in the labor market), limited liability moves the firm away from the first-best and encourages lower safety and higher scale than is optimal. Full liability converges to the first-best in the limit as positive externalities vanishes, because the firm internalizing all the benefits and damages fully aligns its interests with society. This suggests that applying limited liability to all firms is a blunt instrument that can impose too much liability for some firms but not enough for the others compared to what would be the socially

[7]The limited liability rule means that investors are not liable for any amount exceeding what they invest (Easterbrook and Fischel (1985)). Unlimited liability says that unmet obligations of the firm are obligations of the owner, and is always bounded by the debtor's wealth unless one can take a penalty of a pound of flesh as in *The Merchant of Venice* or in a form of indentured servitude or even one's life. All are not accepted in today's civilized world. Even if possible, unlimited liability is still limited more or less. I also include analyses of no liability, which society would likely not make as a rule.

[8]Without liability insurance, for the investor to have a full liability, we would have to have both unlimited liability and what is referred to as "deep pockets," namely that the owner's assets that can be seized to satisfy the obligations are sufficient. If an owner instead has"small pockets, " the obligations are only partially satisfied. A special case of small pockets is "judgment-proof," which means that one has no assets or has only legally protected assets (for instance, a family have all their wealth in a family firm is "judgment-proof").

[9]See Bainbridge and Henderson (2016). For example, Hansmann and Kraakman (1990) claims that "there may be no persuasive reasons to prefer limited liability over a regime of unlimited pro rata shareholder liability for corporate torts." LoPucki (1994) also proposes the abolition of limited liability to achieve "the goal of minimizing the externalization of tort liability."

optimal.

This paper is concerned with the decision making over two dimensions, contrasting with most other theoretical works on tort that only focus on safety incentives. For instance, Shavell (1986) shows that too much risk and too little care would be taken if the injurer has limited assets, and Che and Spier (2008) also claims that injurers strategically using senior bonds to judgment proof themselves will result in less precaution. In the most related work, Hay and Spier (2005) discusses the optimal share of liability of manufacturer and customers but assume perfect competition. Instead, this paper focuses on how the externalities to stakeholders make the equilibrium different from the social optimum.

In Section 2, I start the analysis with the example of the product market in which a single investor chooses safety and scale of the investment. The monopoly firm sets price and the consumers decide the amount to consume depending on the reservation value. I assume that the consumers are not subject to tort damages (for example, an Amazon truck delivering packages hit pedestrians) or they are unaware (for example, the asbestos companies likely hided information for decades). If they knew, the damages could be priced out and resolved by contracts, which is out of the scale of this paper. The contractual cases are interesting, but can be complicated regarding different contracts between the consumers and the manufacturers. Product warranty is an example.[10] In this benchmark, the firm uses rental capital so that limited liability allows only the proceeds in the firm to be reached when damages occur, and the claimants can grab assets outside of the firm under unlimited liability and without any cost.

Following the basic economic analysis, when demand is not perfect elastic, a firm with monopoly power cannot internalize all the benefits of the project and consumers obtain consumer surplus. If a deep-pocket investor has unlimited liability, the result is correct incentives for safety but under provision of quantity: the more inelastic the demand is, the greater the externalization, and the more underinvestment in quantity.[11] (Of course, perfect price discrimination would eliminate consumer

[10]For example, Dybvig and Lutz (1993) and Yang (2010).

[11]This result is not unfamiliar. Indeed, full liability would likely cause inaction when damages are severe enough, and this is probably one reason some jobs are free from liability, such as officials, police officers, volunteers, and charitable organizations, etc.De Geest (2012) uses the multi-task model because these jobs have the character of

surplus and result in efficient choice of quantity. Full liability internalizing all the damages is indeed the first-best.) With sufficient fixed cost, internalizing all the damages results in low quantity ex post, and ex ante the average cost of investment will be higher than the marginal benefit, and starting a firm is not appealing for the investor. However, this inefficiency can be mitigated by limited liability: transferring value from tort claimants to the investor is a way of "subsidizing" the firm and moves the firm towards first-best quantity and has impacts similar to the Ramsey pricing, which suggests that natural monopolist should be subsidized to cover its fixed cost to reach the social optimal. Though this transfer externalizes damages and reduces safety incentives, the loss can be compensated by the gain in terms of social welfare, particularly when the underinvestment in quantity is huge. However, if tort claims are large, the transfer granted by limited liability can over-subsidize the firm and produce too large quantity, which can be socially damaging. In this case, it is possible that the project has little value and shouldn't even get started. Unlimited liability can be tricky when the investor's wealth is not too large: the investor will make a trade-off between whether to invest less and take full liability, or to invest more and have small pockets. When tort is larger and wealth is more constrained, the investor has a tendency of becoming small pockets and likely invests much more in the firm, similar to the strategic judgment proofing, but the difference is that the incentives for safety and scale can go either way.

Section 3 extends the model to multiple firms with Cournot competition. I assume that the homogeneous firms invest in the same technology and each has an investor with the same wealth. I show that full liability moves to the first-best as there are more firms in the market, but this also requires the investors being deep pockets. If the investors instead have small pockets, the equilibrium would be very similar to limited liability: more competition would weaken the positive externalities, and hence capping liability would provide too much compensation for the firm, resulting in overprovision of quantity and underprovision of safety.

These results suggest that liability is not a one-size-fits-all instrument. Therefore, it may be useful to consider other mechanisms that can improve incentives. One remedy is to require mandatory

balancing two negative externalities.

liability insurance, discussed by Shavell (1986, 2004). Following the logic of our framework, requiring full insurance can be damaging since full liability results in under provision of quantity, but requiring a proper level of insurance could make society better off. Shavell (2004) also discusses requiring minimum assets as a cushion, which is more related to Section 4. Section 4 shows that how capital financed by equity (buy capital) can provides better incentives compared to leasing capital or capital financed by non-recourse debt (lease capital). In the benchmark setting, the firm uses leasing capital and only pay tort claimants with the proceeds. Buying capital is desirable under limited liability because it 1) increases available assets in the firm and internalizes more damages, encouraging higher safety and, 2) decreases available assets for investment, encouraging lower quantity. This likely benefits society when there is intense competition. However, unlimited liability will not get any benefits: while it does not change the equilibrium when the investors have small pockets, since placing assets outside and inside of the firm does not change how much the claimants can seize in the model, it discourages deep-pocket investors to start the firm – similar effect of having a fixed cost.

The model can also be applied to other markets and when other stakeholders also share benefits in the firm, as discussed in Section 5. An analogous market is the labor market where employees are the stakeholders. When a firm is the only buyer of labor in the market, the firm has monopsony power. The marginal cost would be greater than the social cost which is also workers' reservation utility. Because of this, the firm does not capture full benefits from the employees working in the firm, and if bearing full liability, would likely operate in a smaller scale. Adding fixed cost only discourages the investor to start the firm in the first place. This may explain smaller firm scales in smaller places when the firms have market power. A classical example would be a coal mining company in a West Virginia town - geographically remote so that finding a substitution is costly. Parallel to the product market, by capping liability it encourages higher labor employed by the firm, which can be regarded as a form of subsidy. This is efficient if the welfare gain from the increased employment and the operation of business covers the loss from dampened safety investment. However, having a lot of competitors is where limited liability is inefficient,

and similarly, the inefficiency can be lowered by requiring some capital installed in the firm as a cushion for tort claimants. In this section, governments and communities as stakeholders are also discussed.

In this framework, a liability rule is said to be "better" if the equilibrium is closer to the first-best (and thus reaches higher social value). This standard welfare approach may not fully align with some notions of fairness mostly related to tort, such as notions concerning punishment and/or compensation as well as the notion follows the logic of "corrective justice" which most of the public, law analysts and policymakers find appealing. [12] However, if a world with individuals who can be investors in some firms and victims in some others, and pursuing social welfare may align well with some notion of "ex ante fairness" where individuals "harmed" in one case can be "compensated" in another.

The reasoning in the law literature for limited liability with respect to tort claims usually follows a premise that investors ought to internalize tort risk.[13] Following this premise, it is argued that limited liability is appropriate for public corporations because forcing to internalize tort risk generates social costs outweighing the social benefits. In particular, one big cost for the joint and several unlimited liability regime is the need to know the available wealth of other shareholders since the most deep-pocket shareholders bear the most tort liability. This could result in large investors heavily monitoring the firm, or no investing, or even highly leveraged firm with a very large amount of secured debt that has priority over tort liability.[14] Then a famous paper Hansmann and Kraakman (1990) proposes pro rata unlimited liability over joint and several rule to solve the information and monitoring problems. Bainbridge and Henderson (2016) disagrees on the rule made by Hansmann and Kraakman (1990) to identify responsible persons, suggesting that it is not practical given the fast changes of the firm's shareholder pool especially with today's finan-

[12]The notion of fairness concerning punishment says that the injurers should pay for any harm that is associated with the act, whereas one concerning compensation says that victims should be made whole. Kaplow and Shavell (2001) has a very detailed discussion of fairness and welfare.

[13]This premise can be justified on the grounds of certain notions of fairness which has a logic of corrective justice: "If A wrongfully injures B, A must pay B for the loss B suffers as a consequence of A's act."See Coleman (1995). Internalizing all the damages can also be justified by a welfare consideration if the firm does not have positive externalities, because the firm would align the interests with society – only a special case in this paper.

[14]Grundfest (1992) argues that there would be "more exotic debt-equity hybrid."

cial market. Leebron (1991) also argues that the collection costs would be too high considering possibly hundreds of thousands of shareholders among whom some are out-of-state, some are off-shore, some own too few shares to worth the effort, and some have small pockets. In addition, investors could always take advantages of other law to evade personal liability, e.g., investing in some real estate or employing independent contractors, etc.[15] Hansmann and Kraakman (1990) has claimed to solve these problems, for example, by proposing mandatory insurance. For closely held firms, unlimited liability is usually claimed appropriate because forcing to internalize tort risk has relatively small social costs.[16] The arguments justifying limited liability usually falls to the investors' preferences considerations (risk-aversion and reputational considerations), and free ride on the capital requirement by the contractual creditors.

2 The Benchmark Model

For now, I consider consumers as the only other stakeholders of the monopoly firm selling a potentially dangerous product, and I look at the implications of allowing the investor to organize the risky activity with different liability rules. In this section, the single firm uses rental capital and hence only loses revenue when compensation has to be made for the tort claimants. In the following sections I increase competition by increasing the number of the firms and I also discuss the impact of rental capital versus purchased capital on the investors' choices of safety and quantity. I will discuss other stakeholders such as employees and government who also capture some of the rents from operation through various institutions.

I begin with a single investor who has wealth W and can choose either to invest in a firm whose products potentially cause damages, or to invest in a safe technology with a payoff $R_f > 0$ (one plus the rate of return) per unit invested.[17] The investor decides the allocation of investment into a

[15] In California, money home mortgages are non-recourse. In agency law, if the tort is committed by an independent contractor, the principal would have limited liability. See Bainbridge and Henderson (2016)

[16] Easterbrook and Fischel (1985).

[17] Wealth W is large enough to avoid discussing financial constraint and trade-offs of investments in operation and safety at the boundary, but it is not unlimited so that the investor is possibly small-pocket.

firm with input s, which produces s units of goods, and the cost of safety of each unit $C(s)$, where s is the probability of each unit of good not causing damages. The care level or safety of the product is $s \in [\underline{s}, 1), (\underline{s} > 0,)$ and $C(s)$ is the unit cost of care satisfying $C'(s) > 0$, $C''(s) > 0$ for $s \in (\underline{s}, 1)$, $C(\underline{s}) = C'(\underline{s}) = 0$ and $\lim_{s\uparrow 1} C'(s) = \infty$. The convexity of cost function implies that the marginal cost of care level is increasing in care. Whatever wealth is not invested in the risky technology is invested in the safe technology with the payoff R_f. Then with probability $1 - s$ each unit of product causes damage $d > 0$ to tort claimants who obtain an effective compensation Λ in total. In a general setting, the damage may be to the user (who has a contractual relationship with the firm and can price out the damages) or to a "third party" (who does not contractually internalize the damages), and can be qualitatively different.

The investor's firm is a monopolist in the product market and sets price p for each unit of good. There is an identical continuum of consumers with measure one, each has an inverse demand function $m(x)$ (satisfying $m'(x) < 0$) which is also the marginal utility of the x^{th} good consumed, excluding any damages not compensated. Then given price p, the inverse demand function $m(x)$, the probability that damages occurs $(1 - s)$, damages per unit of consumption d and the effective liability $\Lambda(s, Q)$, a representative consumer chooses a quantity $Q \in \mathbb{R}_{\geq 0}$ to maximize consumer surplus

$$V_c = \int_{x=0}^{Q} \Big[m(x) - p \Big] dx - \delta(1-s)(Qd - \Lambda(s, Q)) \tag{1}$$

where δ is the fraction of claims associated with using the product. Since all the consumers are identical, V_c is also the total consumer surplus. In equilibrium, $p = m(Q) + \delta(1-s)(d - \partial\Lambda(s, Q)/\partial Q)$, namely, the price should be equal to the marginal benefit of consuming Q goods.

How much liability is associated with a consumer may depend on the property of the product and the consumers' (biased) estimation of tort damage. If $\delta = 0$, consumers do not bear any risk, possibly because of strong information asymmetry as a consequence of manufacturers intentional hiding information or advertising can also be a cause. In Galasso and Luo (2021), the downstream

manufacturers purchase materials from upstream suppliers but sell the products to third parties also belong to this category if only the suppliers are liable.[18] In this case, tort cannot be contracted on the consumers' side to regulate the firm, and hence liability is the only consideration in this framework to internalize the damages. If $\delta = 1$, the liability can be contracted because consumers bear the full damages, and consumers only capture part of the damages when $\delta \in (0,1)$.[19] In this paper I proceed the pure tort case when $\delta = 0$, i.e., the damages are fully borne by a third party who is unknown until the damages occur.[20] In equilibrium, the total consumer surplus is

$$V_c = \int_{x=0}^{Q} \Big[m(x) - m(Q) \Big] dx. \tag{2}$$

The total effective liability $\Lambda(s,Q)$ is a function of care level s and investment Q. Under the limited liability rule, liability is restricted to firm's available assets which is the total revenue from selling the product, i.e. $\Lambda(s,Q) = (Qp) \wedge (Qd)$, where $a \wedge b \equiv \min\{a,b\}$ and $a \vee b \equiv \max\{a,b\}$. The effective liability of a deep-pocket investor under unlimited liability rule is full liability Qd, but that of a small-pocket also depends on the appropriable wealth outside of the firm and is $[R_f(W - (1+C(s))Q)]$. For an investor with limited wealth, the larger the investment in the firm, the more likely the investor is to be small-pocket.[21]

Given the investor's choices of safety and quantity, the social value of the project is

$$S(s,Q) = \int_{x=0}^{Q} \Big[m(x) - (1-s)d \Big] dx - R_f(1+C(s))Q. \tag{3}$$

$$\text{s.t.} \quad Q \geq 0 \tag{4}$$

[18] Hay and Spier (2005) builds a theoretical model to study the shared liability between the suppliers and manufactures when both are liable.

[19] For example, purchasing one more package from Amazon increases the probability that you are the collateral damage of the delivery staff throwing the package to your house, and at the same time increases the chances of delivering trucks passing by your house and causing damages to you, a "double coincidence."

[20] If a consumer of Amazon is hit by Amazon's delivery truck, then the consumer is in principle the third party and his/her consumption likely does not consider the possibility of getting hit.

[21] I don't explicitly model the uncertainty in the litigation process, but that can be implicit embedded in s. If the strict liability rules apply, the firm is responsible whenever an accident occurs; if under the negligence laws, there can be proof of burden and there is uncertainty whether the liability is going into litigation (add reference) especially when the tortious technology is new and not well understood even by the experts. Firms could then perform strategically to affect the litigation. It is an interesting topic but here I assume that the expected liability is foreseeable.

Because we are looking at total surplus, liability is just a transfer from firms to tort claimants and is not in the social value function. With first-best care level s_s^* and investment Q_s^*, the first-best social value is equal to the consumer surplus[22]

$$\begin{aligned} S(s_s^*, Q_s^*) &= \left(\int_{x=0}^{Q_s^*} \Big[m(x) - (1-s_s^*)d \Big] dx - R_f(1+C(s_s^*))Q_s^* \right)^+ \\ &= \left(\int_{x=0}^{Q_s^*} m(x)dx - m(Q_s^*)Q_s^* \right)^+. \end{aligned} \tag{7}$$

With smaller reservation utility, firms tend to produce more[23] and create larger total damages, so that there are more incentives to choose a higher safety level. Similarly, Equation (6) indicates that the risk-neutral investor either should not invest at all when the potential damage is too big or the risk-free investment has a high return, or should keep investing until the marginal return of investment in the firm is exactly equal to the risk-free payoff.

The investor's problem is to choose investment quantity Q and safety s, followed by the consumer's consumption choice. Formally,

$$\max_{s,Q} \left(Qp - (1-s)\Lambda(s,Q) + R_f[W - (1+C(s))Q] \right) \tag{8}$$

$$\text{s.t.} \quad p = m(Q), 0 \le Q \tag{9}$$

$$(1+C(s))Q < W \tag{10}$$

$$\text{and} \quad \underline{s} \le s < 1, \tag{11}$$

[22]The first-best s_s^* and Q_s^* satisfy

$$C'(s_s^*) = \frac{d}{R_f}, \tag{5}$$

$$m(Q_s^*) - (1-s_s^*)d = R_f(1+C(s_s^*)). \tag{6}$$

Equation (7) is derived when replacing the total cost of consumption with $m(Q_s^*)$ (from equation (6)), and the social value equal to the consumer surplus is always non-negative.

[23]$dQ_s^*/dR_f = \frac{1+C(s_s^*)+R_f}{m'(Q_s^*)} < 0$.

where

$$\Lambda(s,Q)=\begin{cases}0, & \text{if no liability}\\ (Qp)\wedge(Qd) & \text{if limited liability}\\ Qd, & \text{unlimited liability with deep pockets}\\ \Big(Qp+R_f[W-Q(1+C(s))]\Big)\wedge(Qd) & \text{unlimited liability with small pockets}\end{cases}$$

It is an immediate observation that investment in safety level is only directly affected by the internalization of damage which is determined by firm's revenue and liability rule. The size of investment is determined by both the externalization of benefit (through consumer surplus) and the internalization of damage (through liability rule).

To ensure the existence and nice features of solution, I make the following assumptions:

Assumption. *A:* $m(Q)$ *is continuous and smooth with* $m(Q)>0, m'(Q)<0$ *and* $\lim_{s\to\infty} m(Q)\leq 0$.

Assumption. *B:* $m(0)>R_f, m'(0)>-\infty$.

Assumption. *C:* $d^2[m(Q)Q]/(dQ)^2 = 2m'(Q)+m''(Q)Q<0$*, or equivalently*

$$2+\frac{d\log m'(Q)}{d\log Q}>0. \tag{12}$$

Assumption A says that the demand curve is downward-sloping, and Assumption B is necessary for the project to be socially valuable because consuming the very first unit should generate more than the reservation payoff. Assumption C ensures unique solution in most cases and will be used in the next section. The following Table 1 compares the investor's choices with different liability rules to the first-best. For more detailed computation see Table 2.

[Table 1 about here.]

PROPOSITION 1. *With full liability, investment in safety reaches first-best but there is always underinvestment in quantity because the investor does not internalize all the benefits. Limited*

Rules	s^*	Q^*	Q_M^*
FL/UL	$=$	$<$	$<$
LL	$<$	$\lessgtr$	$<$
UL-JP	$\lessgtr$	$\lessgtr$	$<$
NL	$<$	$\lessgtr$	$=$

Table 1: ***(Comparison of different liability rules to first-best and pure monopoly)*** *FL/UL: full liability or unlimited liability with deep-pocket investors; LL: limited liability; UL-JP: unlimited liability with small pockets; NL: no liability;* (s^*, Q^*)*: first-best safety and quantity;* Q_M^**: equilibrium quantity when safety has zero cost.*

liability under-provides safety but increases quantity compared to full liability. This can be socially harmful when demand is very elastic and potential tort claims are large.

Proof. With full liability, the first order conditions are

$$(s) \quad C'(s_u^*) = \frac{d}{R_f} \Rightarrow q_u^* = q_s^*$$
$$(Q) \quad \frac{m(Q_u^*) + m'(Q_u^*)Q_u^* - (1 - s_u^*)d}{1 + C(s_u^*)} = R_f$$

Compare the second equation to Equation (6),

$$m(Q_s^*) = m(Q_u^*) + m'(Q_u^*)Q_u^*$$

Since $m'(Q_u^*)Q_u^* < 0$, it must be true that $m(Q_s^*) < m(Q_u^*)$, so that $Q_s^* > Q_u^*$. Similarly, we can prove the proposition with limited liability by comparing the first order conditions. See Appendix A for details. □

Unlimited liability with deep-pocket investors can undermine the incentives for scale, and the inefficiency can be huge when demand is less elastic and the consumer surplus is large, and the firm may not even get started. Limited liability may generate slightly less incentives for safety, but the inefficiency can be compensated by higher quantity and may be socially beneficial.

Limited liability can be regarded as a subsidy from the tort claimants to the investor, which results in lower safety and higher quantity compared to full liability. The incentives for safety is dampened due to the externalization of tort damages, which also mitigates the externalization of benefits and encourages bigger scale. This can be good when too much benefits are externalized due to inelastic demand. However, limited liability is not a one-size-fits-all rule, firstly because there can be over-compensation when the tort claims are large and when demand is very elastic, and secondly, when the investor is small-pocket, the liability is capped even with unlimited liability.

PROPOSITION 2. ***(Unlimited liability with small pockets)*** *If* $\Lambda(s,Q) = Qp + R_f(W - Q(1 + C(s))) < Qd$,

(1) Compared to first-best, there can be under-provision or over-provision of both safety and quantity.

(2) Compared to limited liability, if $Q_j^* < Q_l^*$, *then* $s_j^* > s_l^*$.

(3) The investor is more likely to have small pockets with larger damage d.

(4) $\frac{dQ_j^*}{dW}\frac{ds_j^*}{dW} < 0$. *That is, safety and quantity always move to different directions when wealth changes.*

Proof. See Appendix A2. □

Having small pockets encourages the investor to put more in the firm to reduce available personal assets that can be seized, which means that both safety and quantity can both go up. Relatively speaking, if the demand is more elastic, it is more likely that the small pockets would overinvest in quantity, and vice versa. Contrast to what people may believe that small pockets result in larger social inefficiency compared to deep-pocket investors because they don't internalize damages, the equilibrium of a small-pocket can instead improve social efficiency and may even be close to the social optimum.

PROPOSITION 3. ***(No liability)*** *If* $\Lambda(s,Q) = 0$, *compared to other liability cases, the firm invests least in safety (zero) and most in quantity, which can be greater or less than the first-best quantity*

depending on the demand elasticity.

Proof. the first order conditions are

$$\begin{aligned} (s) \quad & C'(s_0^*)R_f = 0, \text{ and} \\ (Q) \quad & m(Q_0^*) + m'(Q_0^*)Q_0^* = R_f(1 + C(s_0^*)) \end{aligned}$$

$C(s_0^*) = 0$ is immediate, thus $m(Q_0^*) + m'(Q_0^*)Q_0^* = R_f$.

To compared to other liability rules, here we take limited liability as an example. Under limited liability, the first order condition for quantity is $m(Q_l^*) + m'(Q_l^*)Q_l^* = R_f\big(1 + C(s_l^*)/s_l^*$. The inequality $R_f\big(1 + C(s_l^*)\big)/s_l^* > R_f$ indicates $m(Q_l^*) + m'(Q_l^*)Q_l^* > m(Q_0^*) + m'(Q_0^*)Q_0^*$, and by Assumption C we have $Q_l^* < Q_0^*$. The proofs are similar for other liability cases.

Compared to first-best, the condition for the quantity greater than the first-best is

$$e_p(Q_0^*) > \left[1 - R_f/m(Q_s^*)\right]^{-1}.$$

□

Having no liability for damages seems unacceptable, but can happen. If the law system is not well established, powerful firms can get rid of legislation easily. More commonly, tort may involve technologies that are not well understood or the diseases show their symptoms only chronically. The original investors may grab the revenue early and the original firms may shut down before the problem reveals and tracing the parties who should be responsible after decades can be costly. Even if the firm survives, should the tort claimants go after the current firms and investors? What if the firm has already advanced the technology and what if the management has changed? Going after the wrong group does not push incentives to the right direction but can damage the current business.

Considering such situation in which consumers are not the potential victims of tort and the firm is not responsible for any liability, tort is a pure externality and the firm has no incentives

to invest in safety, then damages are most likely to occur. It thus becomes a price-quantity trade-off determined by demand elasticity. It is more damaging when individual damage is large and demand is elastic, and the externalization of tort damage can be mitigated by the externalization of consumer surplus to which extent is somewhat related to demand elasticity.[24] When the demand is not elastic, too large fraction of benefit goes to consumer surplus, and the firm would want to decrease the quantity in investment but the price increases more, which grows the total firm revenue. When tort is large, a socially harmful project can be conducted.

The monopoly quantity: a special case with costless safety

It may be useful to compare the model to the intermediate microeconomics monopoly firm's problem, which is a special case when the firm is always safe. Then we have the following conclusion:

PROPOSITION 4. ***(The monopoly quantity)*** *If $C(s) \equiv 0$, the firm invests in full safety and the same quantity compared to the no liability quantity, and higher quantity compared to the quantities under any other liabilities.*

Proof. Suppose Q_M^* is the equilibrium quantity when the firm is always safe, then the first order condition for quantity is $m(Q_M^*) + m'(Q_M^*)Q_M^* = R_f$. This is the same when there is no liability (i.e., $\Lambda(s,Q) = 0$) in which case the firm chooses minimal safety and the cost of safety is also zero. We can get the rest part of conclusion from Proposition 3. □

The choice of quantity is the highest without any consideration of safety, and is the same level when there is no liability at all. We can think of different liability rules as adding different caps to the liability. With lower caps the investor externalizes more benefits. The result is lower safety incentives and higher quantity incentives. Then there is some level of cap such that it is most socially beneficial along the line – the "second best" choice since we can never reach the first-best

[24] With perfect price discrimination, the firm internalizing all the benefit would inevitably invest more compared to the "social optimum" because there is only negative externality.

because of the market power. The choice of liability should be related to how close they are to this "second best."

The remaining part of this section provides a numerical example of the model and shows equilibria of different liability cases with different parameter values. I also add fixed cost to the model as a comparison. Some observations besides the above propositions are discussed.

An example of linear demand

In this example I assume linear demand. The cost function is assumed so that corner solutions are avoided.

$$
\begin{aligned}
m(Q) &= a - bQ \\
C(s) &= \frac{c}{1-s} + \frac{2\underline{s}c - (1+s)c}{(1-\underline{s})^2}, \quad s \in [\underline{s}, 1)
\end{aligned}
$$

For limited liability, we have the first order conditions

$$
\begin{aligned}
&\text{when } \left(\frac{a-d}{b}\right)^+ \leq Q < \frac{a}{b} \\
&\qquad (s) \quad Q(s) = \frac{a}{b} - \frac{R_f}{b} C'(s) \\
&\qquad\qquad\quad = \frac{a}{b} - \frac{cR_f}{b}\left[\frac{1}{(1-s)^2} - \frac{1}{(1-\underline{s})^2}\right] \\
&\qquad (Q) \quad Q(s) = \frac{a}{2b} - \frac{R_f}{2bs}(1 + C(s)) \\
&\qquad\qquad\quad = \frac{a}{2b} - \frac{cR_f}{2bs}\left[\frac{1}{c} + \frac{1}{1-s} + \frac{2\underline{s} - (1+s)}{(1-\underline{s})^2}\right] \\
&\text{when } 0 < Q \leq \left(\frac{a-d}{b}\right)^- \\
&\qquad\qquad (s) \quad s = 1 - \left[\frac{d}{cR_f} + \frac{1}{(1-\underline{s})^2}\right]^{-\frac{1}{2}} \\
&\qquad\qquad (Q) \quad Q(s) = \frac{a}{2b} - \frac{R_f}{2b}(1 + C(s)) - \frac{(1-s)d}{2b}
\end{aligned}
$$

For unlimited liability, we have

$$\text{when } \left(\left(\frac{a-d}{2b}-\frac{R_f(1+C(s))}{2b}\right)+\sqrt{\left(\frac{a-d}{2b}-\frac{R_f(1+C(s))}{2b}\right)^2+\frac{R_fW}{b}}\right)^+ \le Q < \frac{a}{b}$$

$$(s)\quad Q(s)=\left(\frac{a-sR_fC'(s)}{2b}-\frac{R_f(1+C(s))}{2b}\right)+\sqrt{\left(\frac{a-sR_fC'(s)}{2b}-\frac{R_f(1+C(s))}{2b}\right)^2+\frac{R_fW}{b}}$$

$$(Q)\quad Q(s)=\frac{a}{2b}-\frac{R_f}{2b}(1+C(s))$$

$$\text{when } 0<Q\le\left(\left(\frac{a-d}{2b}-\frac{R_f(1+C(s))}{2b}\right)+\sqrt{\left(\frac{a-d}{2b}-\frac{R_f(1+C(s))}{2b}\right)^2+\frac{R_fW}{b}}\right)^-$$

$$(s)\quad s=1-\left[\frac{d}{cR_f}+\frac{1}{(1-\underline{s})^2}\right]^{-\frac{1}{2}}$$

$$(Q)\quad Q(s)=\frac{a}{2b}-\frac{R_f}{2b}(1+C(s))-\frac{(1-s)d}{2b}$$

The following figures assume some parameter values. Besides the propositions discussed above, there are several observations:

Observation 1. *In Figure 1, when unit damage d is relatively small, full liability can reach the worst social value and even worse than no liability because of inefficiency in under-investment in quantity. Limited liability in both cases are triggered and there is a tendency that as d is smaller, even with limited liability the investor is willing to take the full-liability equilibrium. The incentive is kind of reflected in the unlimited liability case: the investor would have enough assets in equilibrium even though investing more both in quantity and safety would sufficiently enhance social welfare. In these two cases, limited liability is the closest to the first-best.*

Observation 2. *In Figure 2, when unit damage d is relatively large, it is socially better to invest relatively less in quantity and more in safety. Full liability seems to be the best liability rule among the rules shown in the figures when $d = 11$. Both limited liability and unlimited liability tend to have a feature of overinvest in quantity, and limited liability can be bad (but better than no liability at all). The investors in both cases are having small pockets because of the huge damage they cause.*

Observation 3. *Figure 3(a) shows an example that increasing personal wealth can results in full*

liability that is socially less appealing than LL and UL-JP when damage is relatively large. Similarly, with inelastic demand there is less incentives to invest in quantity and results in full liability under the unlimited liability rule and limited liability becomes slight more socially appealing than the unlimited liability rule, as is shown in the last figure in Figure 3(b).

Observation 4. *Figure 4 shows that investment in safety is reduced if it is more expensive to prevent damage. Interestingly, when cost is low, investor under the unlimited liability rule will have small pockets, but the equilibrium is closest to first-best.*

Adding fixed cost

When the firm has fixed cost, average cost is decreasing. In this case the firm would not operate if they have to internalize all the damages, which would result in low quantity ex post. Ex ante, low quantity makes average cost of investment higher than the marginal benefit, and the firm is less likely to start as a consequence. This can be good or bad depending on how big the damages are, shown by the two cases in Figure 5.

Observation 5. *Figure 5(a) The firm would not get started with unlimited liability when fixed cost is sufficiently high. Limited liability is beneficial because it makes it easier for entering and results in positive social value when damages are not too large.*

Observation 6. *Figure 5(b) The firm would not operate, but probably it should not if tort damages are large enough. In this case, limited liability is worse because the huge damages result in negative social value.*

[Figure 1 about here.]

[Figure 2 about here.]

[Figure 3 about here.]

[Figure 4 about here.]

[Figure 5 about here.]

So far I have analyzed a single firm who has market power and can set price for its product. The firm cannot fully internalize all the benefit from operation and the consumers obtain the "triangle" of the demand function. As a consequence, full liability can undermine the incentives to produce and can even halt a beneficial project especially when the demand elasticity is relatively low and the "triangle" is substantial. With fixed cost, the firm is likely not started at all. Limited liability can improve social welfare by increasing quantity produced a large amount, and the social gain from it possibly offsets the social loss from increased probability of damage. The investor and the consumers benefit from limited liability, which hurts the potential tort claimants because their probability of suffering a loss is greater and they are not to be fully compensated. In the next section, I analyze how competition changes the equilibria and the implications of different liability rules.

3 Cournot Competition: a stakeholder example

I show in this section that with more competition, full liability tends to be socially efficient but capped liability (either capped by the limited liability rule or the investors having small pockets) is damaging. This result can also be applied to the labor market and is discussed in more detail in a later section. A monopsony firm in the labor market would similarly result in under-employment because the marginal cost is higher than the reservation cost of hiring. If the market is very competitive instead, there is little externality to the workers and full liability is close to first-best. Competition intensity is measured by the number of firms. Suppose there are N homogeneous firms investing in the same technology, and a representative firm i chooses safety s_i and quantity Q_i. In the simplest case the firm's choice of safety is independent of other firms' choices.[25] The

[25] In some cases, though, a tort litigation would trigger a series of litigation on similar products which are produced by other firms.

social welfare function is the same as (3), and firm i's problem is

$$\max_{s_i,Q_i}\left(Q_i p-(1-s_i)\Lambda(s_i,Q_i)+R_f[W/N-(1+C(s_i))Q_i]\right) \tag{13}$$

$$\text{s.t.}\quad p=m(Q_{-i}+Q_i),0\leq Q_i \tag{14}$$

$$(1+C(s_i))Q_s<W/N \tag{15}$$

$$\text{and}\quad \underline{s}\leq s_i<1 \tag{16}$$

Where

$$\Lambda(s_i,Q_i)=\begin{cases}0, & \text{if no liability}\\ (Q_i p)\wedge(Q_i d) & \text{if limited liability}\\ Q_i d, & \text{unlimited liability with deep pockets}\\ \left(Q_i p+R_f[\frac{W}{N}-Q_i(1+C(s_i))]\right)\wedge(Q_i d) & \text{unlimited liability with small pockets}\end{cases}$$

[Figure 6 about here.]

In equilibrium, all the firms choose the same safety and productivity levels, i.e., $s_i=q,Q_i=I/N$. See Table 3 for detailed computation of the first order conditions. We then have the following proposition:

PROPOSITION 5. ***(Cournot competition)*** *When the number of firms increases to infinity, equilibrium of full liability converges to first-best, whereas limiting liability (either because of the limited liability rule or the investors having small pockets) results in too high investment in quantity and too low investment in safety and is socially inefficient.*

Proof. The first order conditions for full liability are

$$(s_i)\quad s_i^*=s_s^*$$

$$(Q_i)\quad m(Q_u^*)+m'(Q_u^*)Q_u^*/N-(1-s_i^*)d-R_f(1+C(s_i^*))=0.$$

We can compute how Q_u^* changes when the number of firms increases:

$$\frac{dQ}{dN} = \frac{Q/N}{N+1+\frac{m''(Q)Q}{m'(Q)}}.$$

Equation (12) indicates that $\frac{dQ}{dN} > 0$, so the equilibrium quantity is increasing when the number of firms increases. As $N \to \infty$,

$$m(Q_u^*) + m'(Q_u^*)Q_u^*/N - (1-s_i^*)d - R_f(1+C(s_i^*)) \to m(Q_u^*) - (1-s_i^*)d - R_f(1+C(s_i^*))$$

since Q cannot be infinite. When the number of firm increases, while safety is always optimal, the first order condition for quantity is closer to first-best. We can do the same calculation for other liability rules. See Appendix 3. □

[Table 2 about here.]

At extreme, the market becomes perfect competitive and suggests perfect demand elasticity. The efficiency of full liability comes immediately from the fact that the firm internalizing all the benefits and costs fully aligns its interest with that of society. However, competition is bad when liability is not fully internalized. Both limited liability and unlimited liability with small pockets would result in inefficiency of overinvestment in quantity and unerinvestment in safety, deviating from the first-best. In order for full unlimited liability under intense competition to work, one also has to make sure that the investors have enough assets to cover all the liability, otherwise competition would encourage incentives of evading liability by investing more and leaving less outside the firm and is inefficient.

4 Rental Capital or Purchased Capital

In the previous section, the only assets in the firm available to pay tort claimants are the proceeds from sale. This is true if the firm rents capital or uses debt to finance its capital and can pledge

the capital to ensure that the repayment to the lender has higher priority in resolution. When tort occurs, the firm has to return the rental capital or transfer the capital to the lender so that it is out of reach of the tort claimants. This section discusses the situation in which the firm can only buy capital to produce. This may happen when the operating capital is very specialized (especially for new technology) and the firm may not be able to rent existing capital or find lenders to provide funds. It may also come from capital requirement for regulatory purpose or requirement by other contractual creditors as a cushion. In this section I assume proportional capital installment, namely, each unit of production requires k units of capital. Capital does not depreciate when tort occurs as a simplification in the model, but if it does (probably because capital is firm specific and should be liquidated with loss), the loss would make rental price higher if priced out. With firm purchasing capital, this depreciation is fully absorbed by the tort claimants in the limited liability case, and can be partially or fully borne by the investor under unlimited liability. I focus on the case in which there is no depreciation and the firm can sell the capital at the original value kQ_i.

The social welfare function is the same as (3). Firm Q's problem is

$$\begin{aligned}
&\max_{s_i,Q_i}\left(Q_i p-(1-s_i)\Lambda(s_i,Q_i)+R_f[W/N-(1+k+C(s_i))Q_i]\right)\\
&\quad\text{s.t.}\quad p=m(Q_{-i}+Q_i),0\le Q_i\\
&\qquad\quad kQ_i+(1+C(s_i))Q_i<W/N\\
&\quad\text{and}\quad \underline{s}\le s_i<1
\end{aligned}$$

Where

$$\Lambda(s_i,Q_i)=\begin{cases}
0, & \text{if no liability}\\
[Q_i(p+k)]\wedge(Q_i d) & \text{if limited liability}\\
Q_i d, & \text{unlimited liability with deep pockets}\\
\left(Q_i p+Q_i k+R_f[\frac{W}{NQ_i}-Q_i(k+1+C(s_i))]\right)\wedge(Q_i d) & \text{unlimited liability with small pockets}
\end{cases}$$

[Figure 7 about here.]

PROPOSITION 6. ***(Buying capital)*** *If capital is purchased, more damages are internalized under limited liability so that equilibrium is pushed towards first-best with sufficient competition. Capital installment does not change the equilibria of unlimited liability with small pockets, but pushes the equilibria of deep pockets away from the first-best and may even turn down the investment.*

Proof. Limited Liability requires $m(Q_l^*) < d - k$ and the first order conditions

$$\begin{aligned} &(s_i) \quad m(Q_l^*) = -k + R_f C'(s_l^*) \\ &(Q_i) \quad m(Q_l^*) + m'(Q_l^*)Q_l^*/N = -k + R_f(1 + k + C(s_l^*))/s_l^*. \end{aligned}$$

Detailed computation see Table 3. It is easy to prove that $\frac{dQ_l^*}{dk} > 0$ for the first equation and $\frac{dQ_l^*}{dk} < 0$ for the second, hence the function $Q_l^*(s_l^*)$ shifts upward for the first equation, and shifts downward for the second as k increases, resulting in lower s_l^* and higher Q_l^*. With full liability, we have

$$\begin{aligned} &(s_i) \quad s_i^* = s_s^* \\ &(Q_i) \quad m(Q_u^*) + m'(Q_u^*)Q_u^*/N - (1 - s_i^*)d - R_f(1 + k + C(s_i^*)). \end{aligned}$$

Since $\frac{dQ_l^*}{dk} < 0$ for the second equation, the equilibrium safety does not change but the quantity drops as k increases. For unlimited liability:

$$\begin{aligned} &\text{if } m(Q_u^*) + R_f(W/Q_u^* - 1 - k - C(s_u^*)) < d - k \\ &\qquad (s_i) \quad m(Q_u^*) + k + R_f(W/Q_u^* - 1 - k - C(s_u^*)) - s_u^* R_f C'(s_u^*) \\ &\qquad (Q_i) \quad m(Q_u^*) + m'(Q_u^*)Q_u^*/N + k - R_f(1 + k + C(s_u^*)) \end{aligned}$$

When R_f is close to 1, the problem is close to the problem with rental capital. It is true that capital in the firm does not earn risk-free interest outside of the firm, but if the difference is negligible, capital installment would only act as a fixed cost and shut down the firm if sufficient. □

With unlimited liability, requiring (purchased) capital in effect adds overhead, either a fixed number or in this setting a proportional cost, to the investor. As shown in Figure 7 (2), the deep pockets still invest in first-best safety but underinvestment in quantity. It does not change the equilibria for small pockets. However, requiring capital internalizes damages for limited liability, increasing incentives for safety and at the same time decreasing incentives for quantity and improves social welfare particularly when there is more competition.

Notice that, capital installment also moves the budget down, and if sufficient enough, makes some equilibria above the budget line infeasible. In this case, investments in safety and quantity for both limited liability and the small pockets may go down and move along the budget line. Along the line, the two equilibria coincide because the small pockets are judgment-proof and do not have assets outside of the firm, and act as if they had limited liability.

5 Other Stakeholders

Previously, consumers are the only stakeholders of the firm. The inefficiency of underinvestment in quantity is a result of firm's market power in the product market that externalizes benefits, and as discussed before, limited liability has a flavor of Ramsey pricing and may enhance social welfare by "subsidizing" the firm through reducing the liability from damages. This is also true if the firm has other stakeholders. For example, large corporations usually have big impact on communities where the firms located in. They create jobs, provide investment opportunities, safety, unique community identity, economic health and development, etc. Much of the characteristics are valuable but not captured by the profits. Another aspect is the discrepancy of management interest and shareholder interest, which are not necessarily align because of separation of ownership and control. Management nowadays usually do not take full liability or even exempt from liabilities on the consequences of decision making, except for fraudulent conveyance and breach of duty. Part of the reason is that they do not capture all the benefits in the firm and therefore bearing full liability would probably result in too conservative investment strategy and may let go of profitable

investment opportunities. In this section, I discuss two other firm's stakeholders: governments and employees.

5.1 Governments as a stakeholders: taxation

Governments are considered as a major stakeholder of a corporation because they collect corporate income taxes from the firm, payroll taxes from the employees, as well as other taxes (sales taxes, etc.). In some states, certain corporations also pay franchise taxes for the right to be chartered. With higher taxes, the firm externalizes larger proportion of benefits. A lump sum tax such as a franchise tax would be similar to a fixed cost to a firm. It may not distort incentives once the firm is established, but it makes it less attractive to start the firm in the first place. I assume unit tax with tax rate $\tau \in [0,1)$ in this example, then the actual revenue obtained is $(1-\tau)p$ per unit of good sold. Figure 8 shows an example when per unit taxes are 0, 0.25 and 0.5, respectively.

[Figure 8 about here.]

PROPOSITION 7. ***(Taxation)*** *The solid shapes represent the same as in Figure 6 in which there is no tax. The hollow shapes represent the relevant liability rules with unit tax .25, and the crossed hollow shapes represent the relevant liability rules with unit tax .5. With full liability, taxation makes investing less attractive and the investment can be blocked in the first place (See the red crossed hollow dots drop to 0 quantity). When liability is capped either by limited liability rules (the pink squares) or by investors having small pockets (the blues), higher taxation results in lower quantity and safety following the black dashed arrows.*

Proof. See Appendix A4. □

Higher tax rate would externalize more benefits and results in lower quantity for all cases. When liability is capped, taxation results in less assets in the firm to compensate tort claimants. This also discourages safety incentives. As shown in Figure 8, taxation is bad socially with deep pockets because the firm may not start. Yet it is not necessarily bad socially when liability is capped, especially when there is more competition. With intense competition, quantity can be way

too high, and taxation may affect quantity more than safety and improve overall efficiency. With proper taxation, equilibrium can even be close to first-best when the investors with small pockets have unlimited liability. In this specific example, $\tau = .25$ and $N = 1$ reaches an equilibrium that is almost first-best.

5.2 Employees as stakeholders

In the model I focus on the product market, but we can have similar analysis on the labor market. If the firm has monopsony power in the labor market, then the firm does not capture the full benefits of people working in the firm and will tend to operate in a smaller scale. Monopsony is not unusual in the U.S. labor market. A typical example is a mining town in the mountains, where it is remote and has only few mining employers. If the firm is the only employee, the marginal cost is bigger than the workers' reservation utility, because to hire one more worker the wage has to increase for every worker that the firm hires. This probably partly explains small scales of firms in small places where firms have monopsony/monopoly power in the labor market as well as other factor markets. Similar to the conclusion before, if there is also fixed cost, the firm probably would not start in the first place. Beneficiaries have long advocated for unionization and increased wages to a level comparable to a competitive outcome to achieve a more "equitable economy," and the thought can be traced back to as early as Robinson (1933), but this would externalize more benefits and consequently reduces demand for labor and social welfare.

Our model suggests an analysis parallel to the product market analysis: limited liability would mitigate the inefficiency of not internalizing all the benefits, and social efficiency in quantity may be improved. Increasing competition only improves social welfare when the firm bears full liability, which requires unlimited liability and investors have deep pockets. Increasing competition will result in too low safety and too high quantity when liability is capped. With proper capital requirement, the equilibrium under limited liability may be drawn towards the first-best, but that does not work for unlimited liability.

6 Conclusion

This paper provides a theoretical framework to study choice of safety and scale under limited and unlimited liability rules. When the firm's other stakeholders obtain large benefits from the firm, full liability results in under provision of quantity. I mainly focus on the product market in which such inefficiency is a result of big consumer surplus when the firm has monopoly power and faces a less elastic demand. We can also extend the discussion when the firm has other stakeholders, such as communities, governments, and when the firm has market power in other markets such as the labor market. Limited liability mitigates the inefficiency caused by externalization of benefits, because it reduces the damages taken by the investors as a means of subsidy.

With intensified competition, firms capture higher fraction of benefits would have quantities converge to the first-best under full liability, but that also requires the investors to be deep-pocket. If the investors have small pockets, they would overinvest in quantity and underinvest in safety, which has a similar effect to limited liability. However, one advantage of limited liability is that requiring proper amount of assets in the firm or require proper amount of insurance as a buffer can move the firm towards the first-best.

So far I have only studied the single investor's problem. Even when competition is discussed, each firm only has one investor because the intention is to focus on the effect of reduced market power. It would be interesting to study multiple investors in the firm, namely, shareholders and bondholders, and to answer questions such as who should be in control and how they are sharing liabilities. In that setting, priority order of different claimants, monitoring and other policy can also be discussed.

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Appendix A1.

The firm's problem is to maximize

$$(1-s)Q\Big(m(Q)-d\Big)^{+}+sQm(Q)+R_fQ[W/Q-\big(1+C(s)\big)]$$

The first order conditions are

$$\begin{aligned}
&\text{if } m(Q)^{+}\leq d\\
&\qquad (s)\quad m(Q)=R_fC'(s)\\
&\qquad (Q)\quad m(Q)+m'(Q)Q=R_f(1+C(s))/s\\
&\text{if } m(Q)^{-}\geq d\\
&\qquad (s)\quad d=R_fC'(s)\\
&\qquad (Q)\quad m(Q)+m'(Q)Q-R_f(1+C(s))-(1-s)d=0
\end{aligned}$$

When $m(Q)<d$,

$$m(Q_l^*)=\frac{R_f(1+C(s_l^*))/s_l^*}{1-1/e_p(Q_l^*)} \tag{17}$$

is derived from the first order condition above.

When $e_p(Q_l^*)>\big[1-\frac{R_f\left(1+C(s_l^*)\right)}{s_l^*m(Q_s^*)}\big]^{-1}$, we have $Q_l^*<Q^*$.

Appendix A2.

The firm's problem is to maximize

$$(1-s)Q\Big(m(Q)+R_f[W/Q-\big(1+C(s)\big)]-d\Big)^{+}+sQ\Big(m(Q)+R_f[W/Q-\big(1+C(s)\big)]\Big)$$

The first order conditions are

$$
\begin{aligned}
&\text{if } m(Q)+R_f(W/Q-1-C(s))<d \\
&\qquad (s) \quad m(Q)+R_f(W/Q-1-C(s))-sR_fC'(s)=0 \\
&\qquad (Q) \quad m(Q)+m'(Q)Q-R_f(1+C(s))=0 \\
&\text{if } m(Q)+R_f(W/Q-1-C(s))\geq d \\
&\qquad (s) \quad d-R_fC'(s)=0 \\
&\qquad (Q) \quad m(Q)+m'(Q)Q-R_f(1+C(s))-(1-s)d=0
\end{aligned}
$$

Since $m(Q_j^*)+R_f(W/Q_j^*-1-C(s_j^*))<d, C'(s_j^*)s_j^*<C'(s_s^*)$ indicating that $s_j^* \lesseqgtr s_s^*$. Then $m(Q_j^*)+m'(Q_j^*)Q_j^*<m(Q_s^*)$ and Q_j^* can be either greater or less than Q_s^*.

For the second statement, if $Q_j^*<Q_l^*$, then $m(Q_j^*)+R_f(W/Q_j^*-1-C(s_j^*))>m(Q_l^*)$, indicating that $C'(s_j^*)>C'(s_l^*)$, thus $s_j^*>s_l^*$.

For (3), d does not enter the first order conditions when $m(Q_j^*)+R_f(W/Q_j^*-1-C(s_j^*))<d$. It affects the threshold of being judgment-proof. However, It is uncertain how W affects the threshold because s and Q would also change.

Specifically, when $W\uparrow$, $s\downarrow$ indicates that $Q\uparrow$, otherwise the FOC for s does not hold equal. This can also be confirmed by the FOC for Q following assumption C. If $s\uparrow$ instead, the FOC for Q suggests that $Q\downarrow$.

Appendix A3.

We can do the same calculation for limited liability: when $m(Q_l^*) < d$,

$$
\begin{aligned}
(s_i) \quad & m(Q_l^*) - R_f C'(s_l^*) \\
(Q_i) \quad & m(Q_l^*) + m'(Q_l^*)Q_l^*/N - R_f(1+C(s_l^*))/s_l^*. \\
\frac{dQ}{dN} &= \frac{I/N}{N+1+\frac{m''(Q)Q}{m'(Q)}+\frac{m'(Q)Q}{sR_fC''(s)}} \\
\frac{ds}{dN} &= \frac{m'(Q)}{R_fC''(s)}\frac{dQ}{dN}
\end{aligned}
$$

When N is sufficiently large, $\frac{dQ}{dN} > 0$ and $\frac{ds}{dN} < 0$. This is similar for unlimited liability with judgment-proof investors.

Appendix A4.

The investor's problem is

$$\max_{s_i,Q_i}\left(Q_i\left[(1-\tau)p-(1-s_i)\Lambda(s_i,Q_i)\right]+R_f[W/N-(1+C(s_i))Q_i]\right) \tag{18}$$

$$\text{s.t.} \quad p = m(Q_{-i}+Q_i), 0 \le Q_i \tag{19}$$

$$(1+C(s_i))Q_s < W/N \tag{20}$$

$$\text{and} \quad \underline{s} \le s_i < 1 \tag{21}$$

Where

$$\Lambda(s,Q)=\begin{cases}0, & \text{if no liability}\\ (1-\tau)p\wedge d & \text{if limited liability}\\ d, & \text{unlimited liability (deep-pocket)}\\ \Big((1-\tau)p+R_f[\frac{W}{NQ_i}-(1+C(s_i))]\Big)\wedge d & \text{unlimited liability (judgment-proof)}\end{cases}$$

Limited Liability requires $m(Q_l^*)<d/(1-\tau)$ and the first order conditions

$$\begin{aligned}&(s_i)\quad m(Q_l^*)=\frac{R_f}{1-\tau}C'(s_l^*)\\ &(Q_i)\quad m(Q_l^*)+m'(Q_l^*)Q_l^*/N=\frac{R_f}{1-\tau}(1+C(s_l^*))/s_l^*.\end{aligned}$$

It is easy to prove that $\frac{dQ_l^*}{d\tau}<0$ for both the first and the second equation, hence both the functions $Q_l^*(s_l^*)$ shift down, resulting in lower s_l^* and Q_l^*. With full liability, we have

$$\begin{aligned}&(s_i)\quad s_i^*=s_s^*\\ &(Q_i)\quad m(Q_u^*)+m'(Q_u^*)Q_u^*/N=(1-s_i^*)\frac{d}{1-\tau}+\frac{R_f}{1-\tau}(1+C(s_i^*)).\end{aligned}$$

Since $\frac{dQ_l^*}{d\tau}<0$ for the second equation, the equilibrium safety does not change but the quantity drops as τ increases. For unlimited liability:

$$\begin{aligned}&\text{if } m(Q_u^*)+\frac{R_f}{1-\tau}(W/Q_u^*-1-C(s_u^*))<d/(1-\tau)\\ &\qquad(s_i)\quad m(Q_u^*)+\frac{R_f}{1-\tau}(W/Q_u^*-1-C(s_u^*))=s_u^*\frac{R_f}{1-\tau}C'(s_u^*)\\ &\qquad(Q_i)\quad m(Q_u^*)+m'(Q_u^*)Q_u^*/N=\frac{R_f}{1-\tau}(1+C(s_u^*))\end{aligned}$$

$\frac{dQ_u^*}{d\tau} < 0$ is easy to prove for the second equation. From the first equation, compute

$$\frac{dQ_u^*}{d\tau} = \frac{m(Q_u^*)/(1-\tau)}{m'(Q_u^*) - \frac{R_f W}{(1-\tau)(Q_u^*)^2}} < 0.$$

Therefore, both quantity and safety go down.

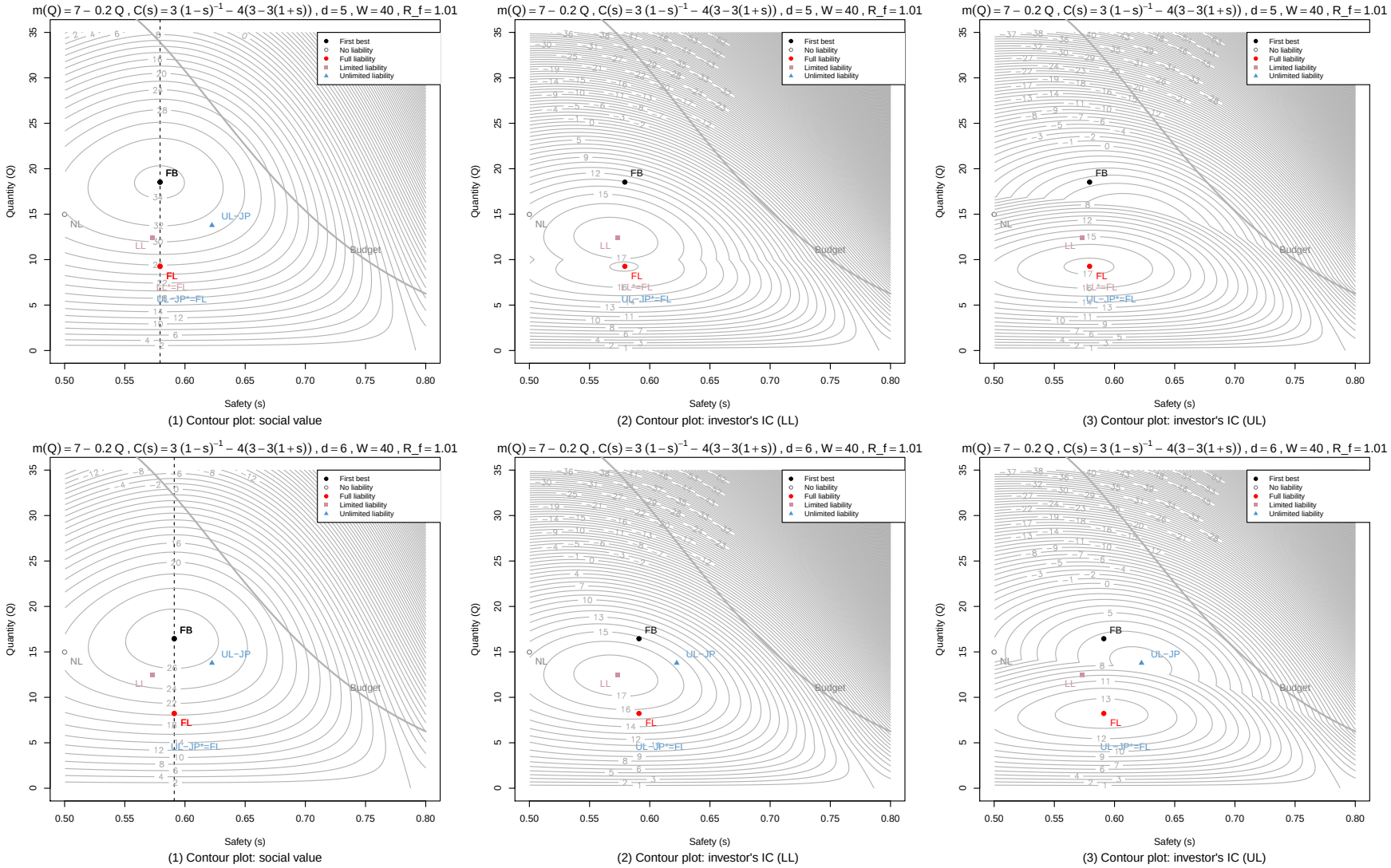


Figure 1: ***(Unit damage** d **is relatively small.)*** *The x axis is the safety level, y axis the quantity level. The dots of different shapes and colors represent the equilibrium of different liability rules. Each row of plots have the same parameter values, and the contour plots are showing the indifference curves of social value, investor's value with limited liability and investor's value with unlimited liability, respectively.* ***Budget line:*** *the grey line labeled "Budget" is the frontier of the feasible investment set. The firms can invest less in quantity and safety and fall below the budget line.*

In these two examples, unit damages are relatively small ($d = 5$ and $d = 6$), full liability can reach the worst social value and even worse than no liability because of inefficiency in under-investment in quantity. Limited liability in both cases are triggered and there is a tendency that as damage is smaller, even with limited liability the investor is willing to take the full-liability equilibrium. The incentive is kind of reflected in the unlimited liability case: the investor would have enough assets in equilibrium even though investing more both in quantity and safety would sufficiently enhance social welfare. In these cases, limited liability is the closest to the first-best.

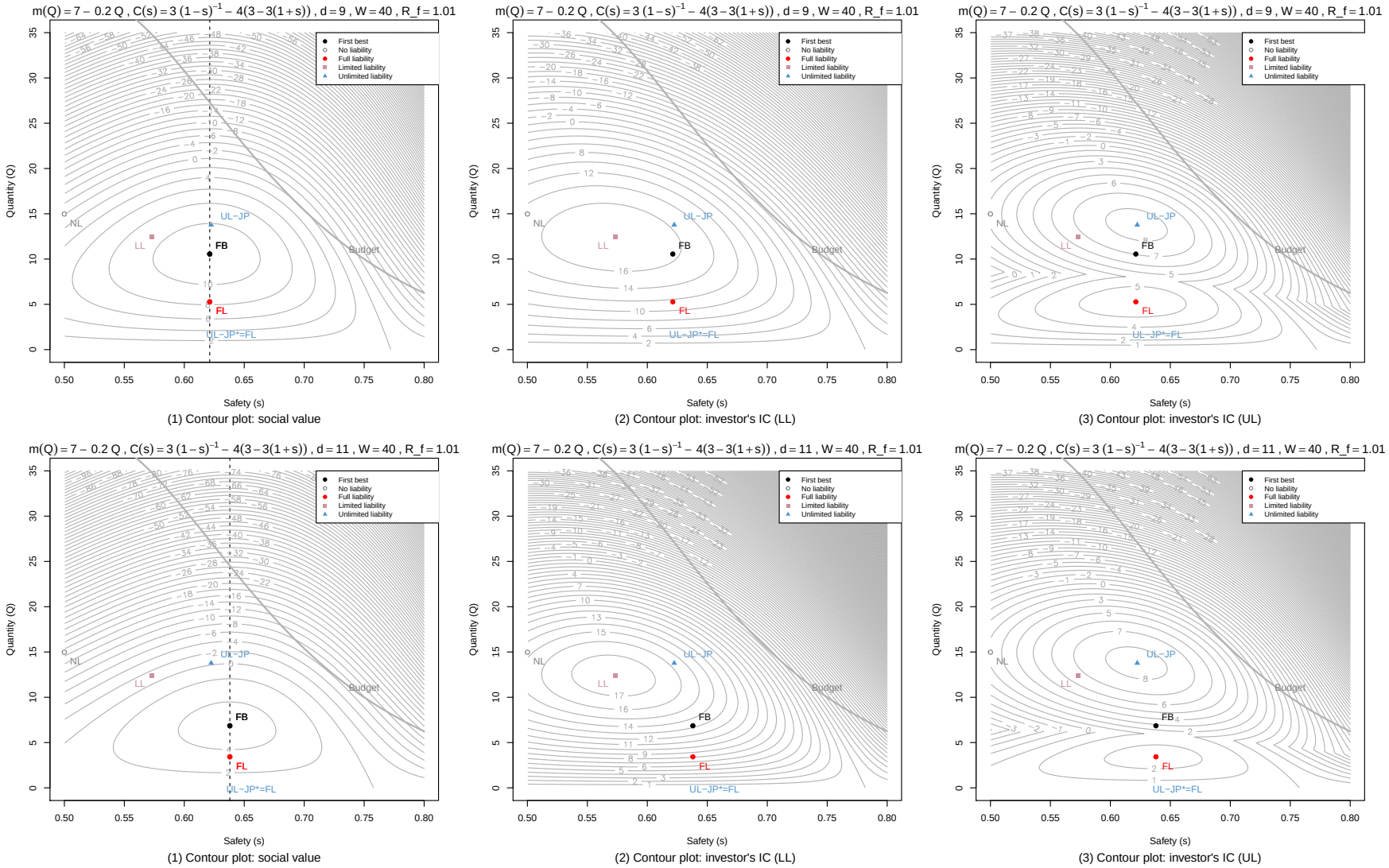
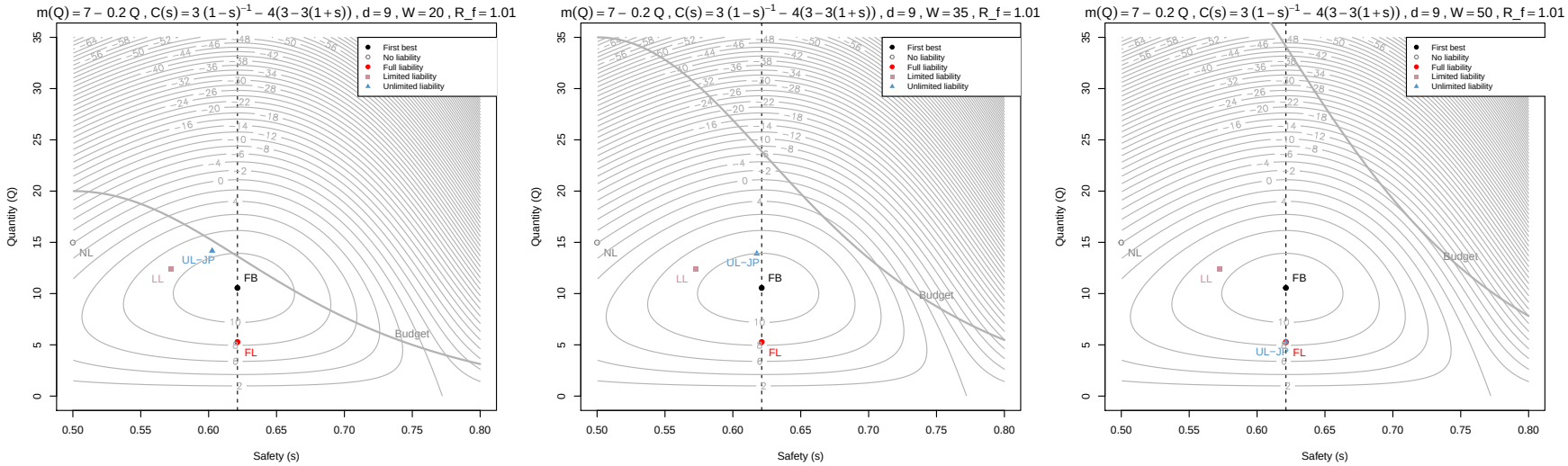
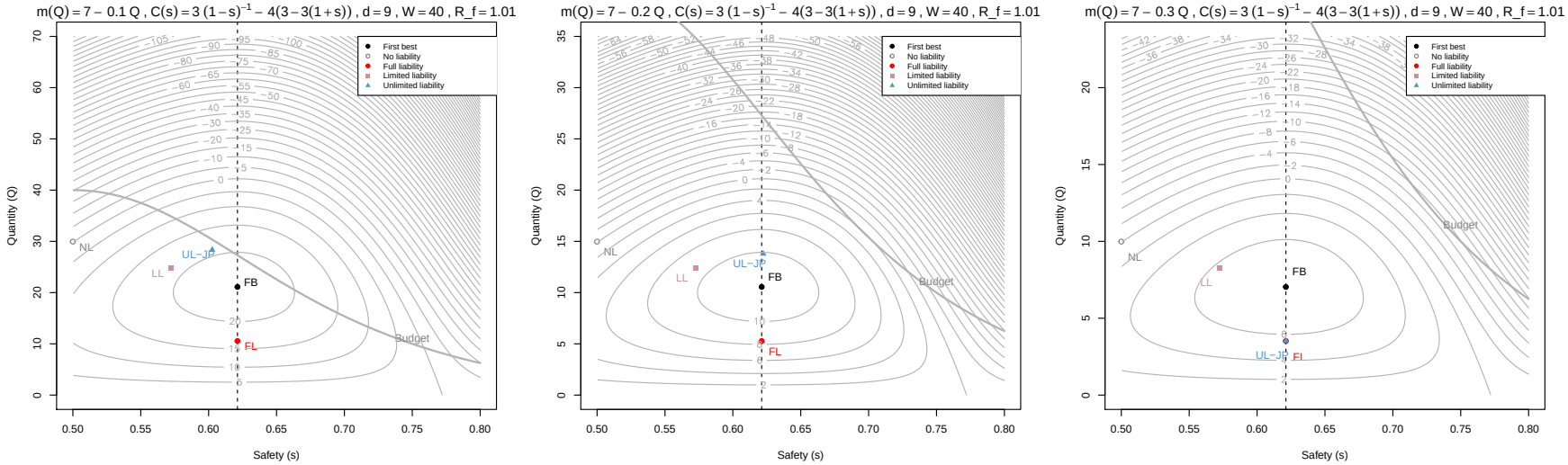


Figure 2: ***(Unit damage** d **is relatively large.)*** *As a comparison to Figure 1, now the unit damage is large. We can observe that it is socially better to invest relatively less in quantity and more in safety. Full liability seems to be the best liability rule among the rules shown in the figures when* $d = 11$. *Both limited liability and unlimited liability tend to have a feature of over-invest in quantity, and limited liability can be bad (but better than no liability at all). The investors in both cases have small pockets because the damage is huge and the profit per unit good is low due to the large quantity.*

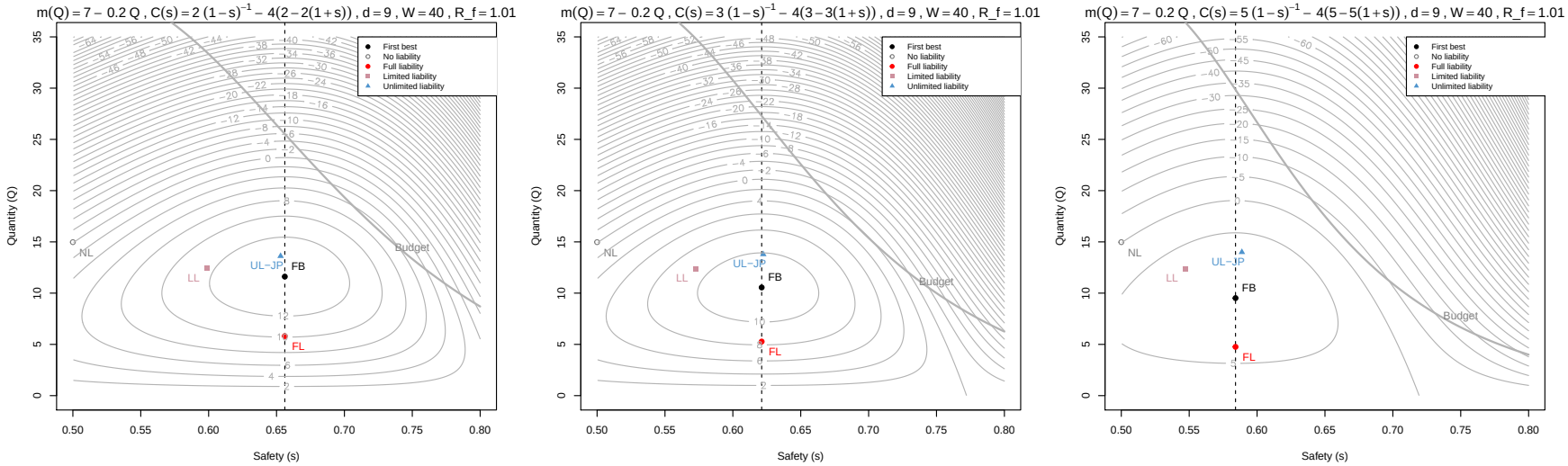


(a) Change W. *In these three plots, everything is fixed except for W. The plots show that unlimited liability*



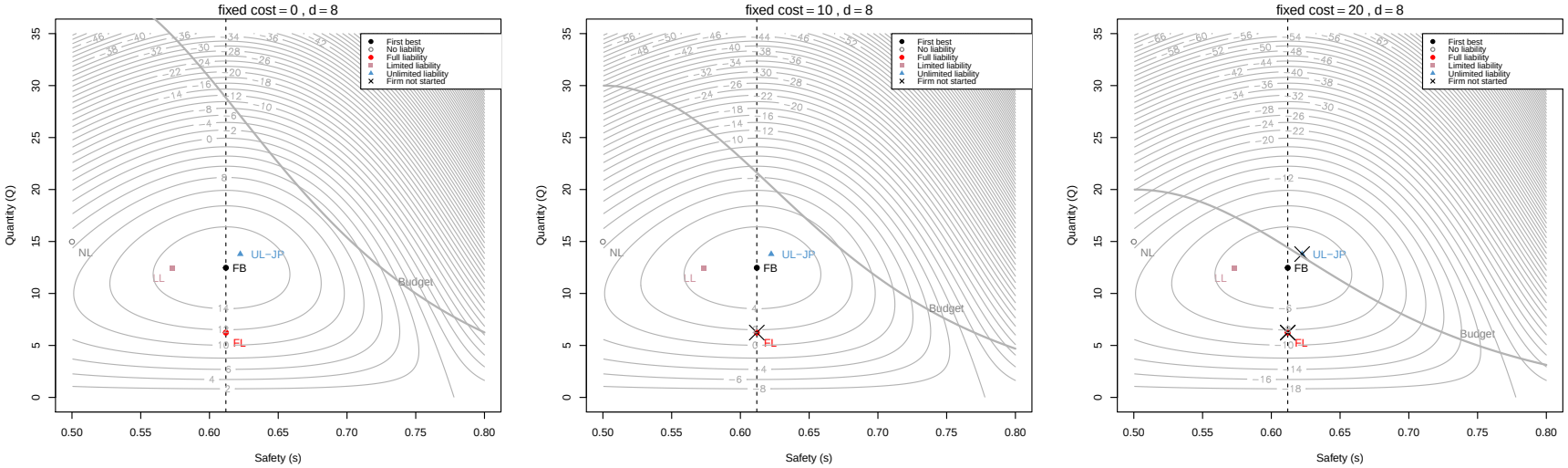
(b) Change demand elasticity. *When demand become more inelastic, quantities decrease for every equilibrium. In this example, social values for all liability rules also decrease.*

Figure 3

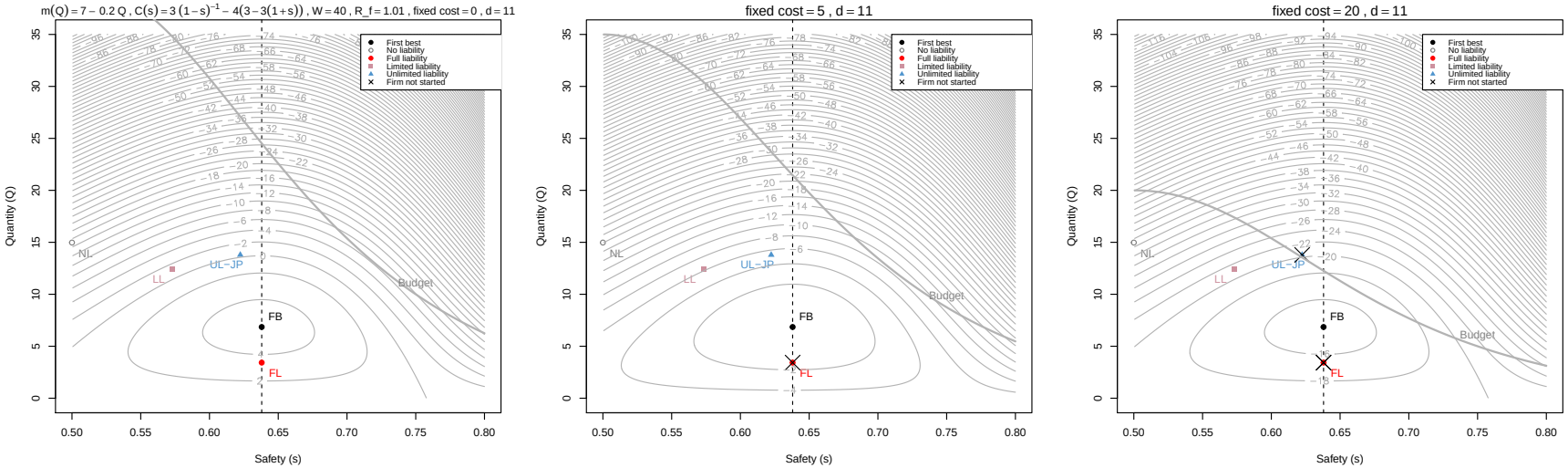


(a)

Figure 4: ***Change safety cost.*** *The x axis is the safety level, y axis the quantity level. The dots of different shapes and colors represent the equilibrium of different liability rules. All the parameter values are fixed except for the safety cost. The plots show that when it is more expensive to prevent damages, there tends to be less investment in safety for every case.*



(a) Change fixed cost when damage are small. *The x axis is the safety level, y axis the quantity level. The dots of different shapes and colors represent the equilibrium of different liability rules and the black crosses are indicating the firm has negative value and would not get started, which happens with unlimited liability and when fixed cost is sufficient. Limited liability is beneficial because it makes it easier for entering and results in positive social value.*



(b) Change fixed cost when damages are large. *The firm would not get started with full liability. In this case, limited liability is worse because the damages are huge which results in negative social value.*

Figure 5

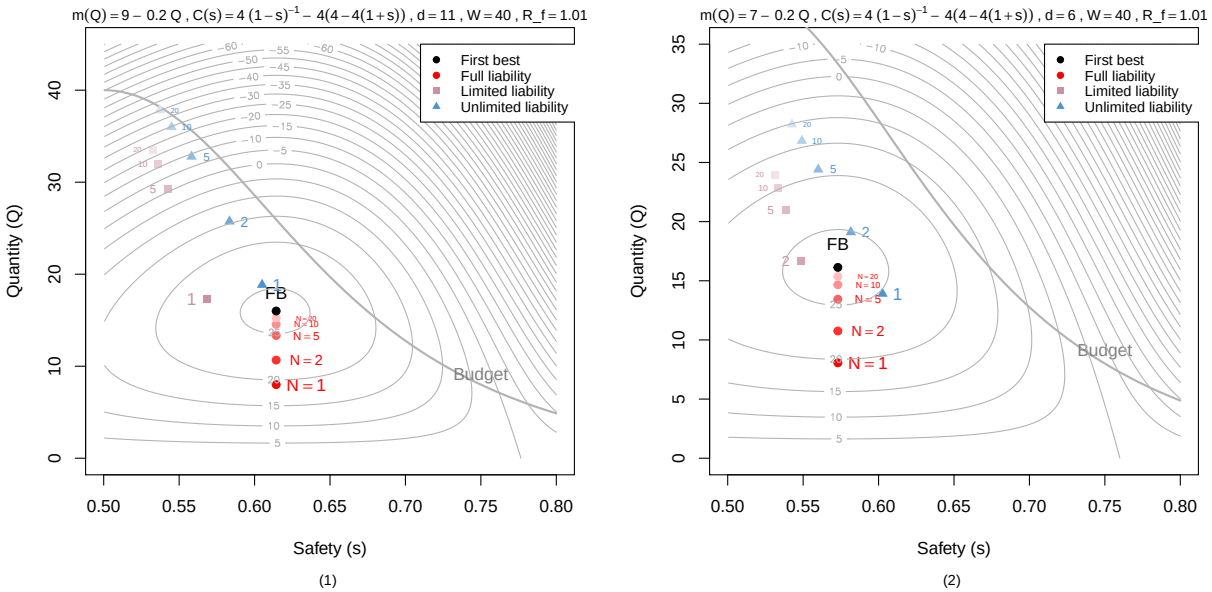


Figure 6: ***(Cournot competition)*** *The x axis is the safety level, y axis the quantity level. The dots of different shapes and colors represent the equilibrium of different liability rules. When the number of firms increases (N =1,2,5,10,20 shown in the graph with the same shapes and colors but reduced saturation), equilibrium of full liability converges to first-best (red dots), whereas limiting liability (either because of the limited liability rule indicated by the pink squares or the investors having small pockets indicated by the blue triangles) deviates from first-best.*

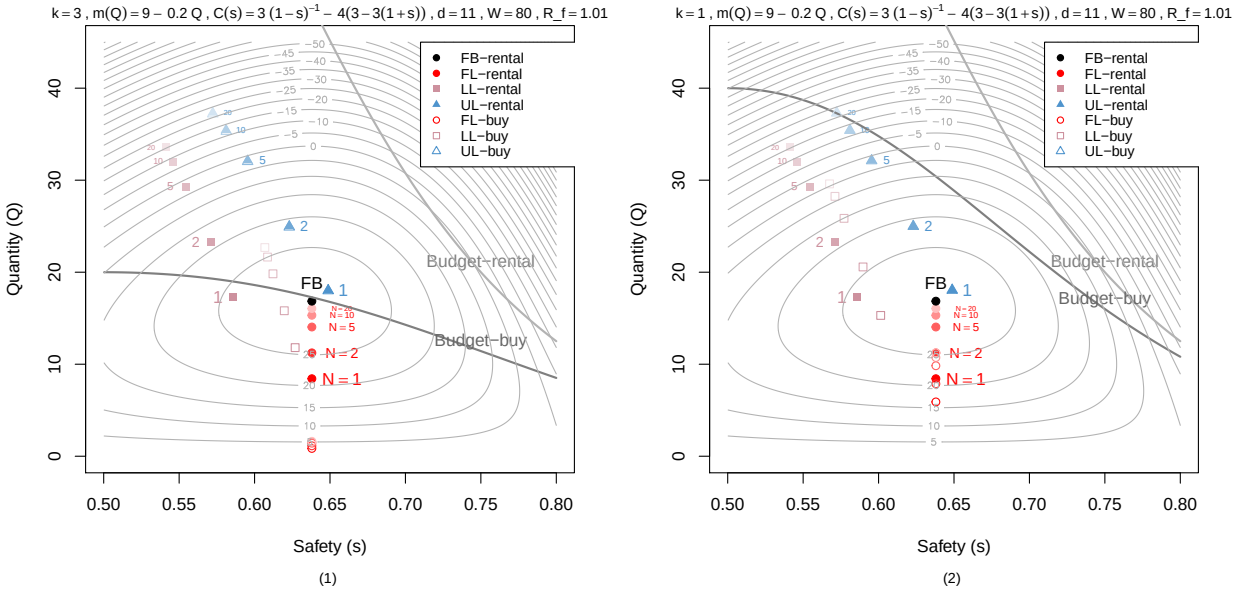


Figure 7: ***(Lease versus buy)*** *The solid shapes represent the same as in Figure 6. The hollow shapes represent the relevant liability rules with purchased capital instead of rental capital. The plot shows that requiring purchased capital may push the equilibrium to first-best for limited liability shown by the hollow pink dots, but either do not change incentives for small pockets under unlimited liability (depicted by the blue hollow triangles) or tend to be inefficient when the investors are deep-pocket (depicted by the red hollow dots).*

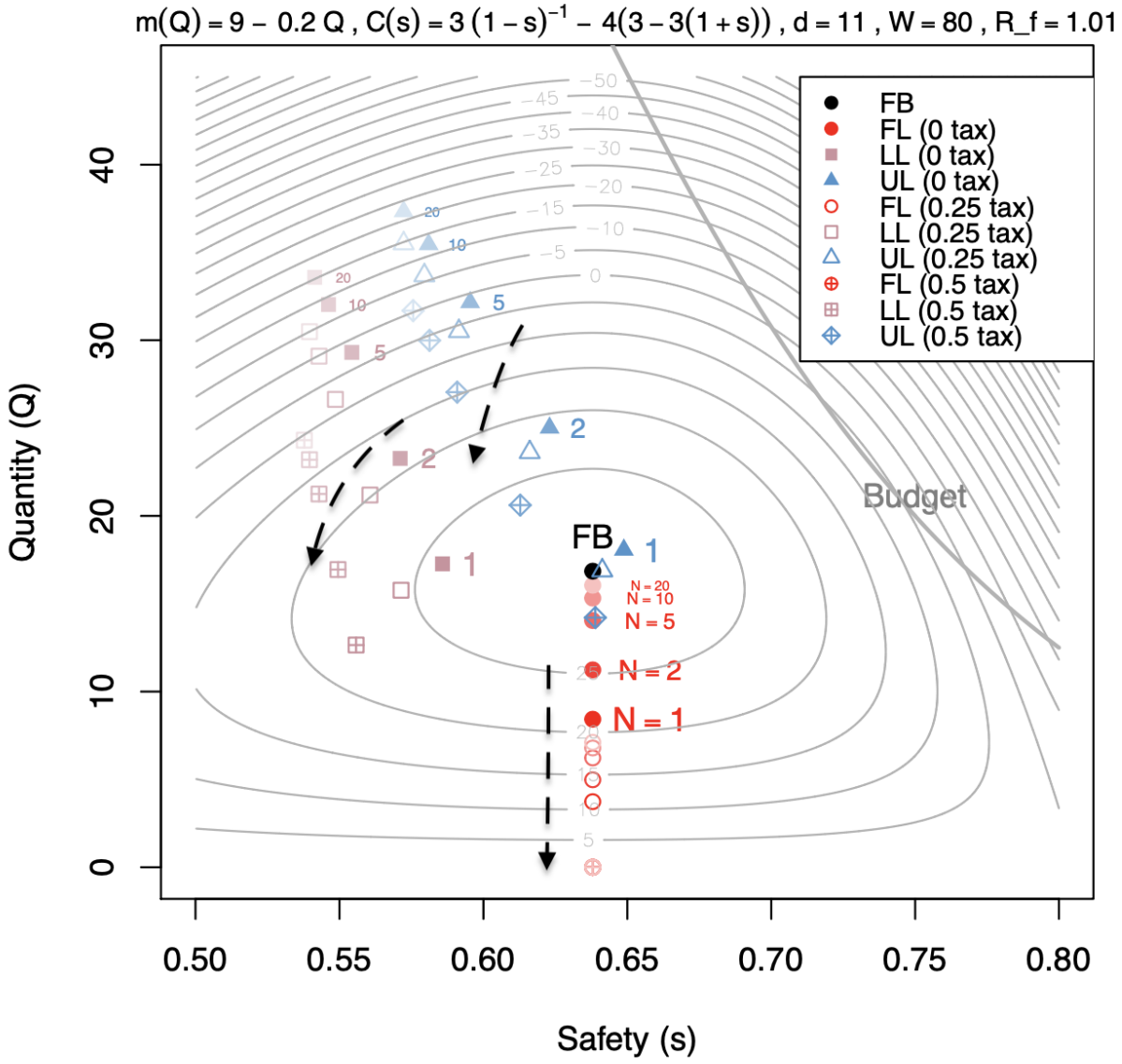


Figure 8: ***(Taxation)*** *Full liability results in underinvest in quantity when there is taxation, and the firm would not start in the first place. When liability is capped, increasing unit taxes undermines both quantity and quality incentives.*

(1)	(2)	(3)	(4)	(5)	(6)	(7)
rule	$\Lambda(s,Q)$	FOC:(s)	FOC:(Q)	Firm started ($Q^*>0$)	$s^*<s_s^*$	$Q^*>Q_s^*$
FB	-	$C'(s_s^*)=\frac{d}{R_f}$	$m(Q_s^*)\leq R_f\big(1+C(s_s^*)\big)$ $+(1-s_s^*)d$	$m(0)>R_f\big(1+C(s_s^*)\big)$ $+(1-s_s^*)d$	(always $=$)	(always $=$)
FL/UL	Qd	$C'(s_u^*)=\frac{d}{R_f}$	$m(Q_u^*)+m'(Q_u^*)Q_u^*\leq$ $R_f\big(1+C(s_u^*)\big)+(1-s_u^*)d$	$m(0)>R_f\big(1+C(s_u^*)\big)$	(always $=$)	(always $<$)
LL	$Qp(<Qd)$	$C'(s_l^*)=\frac{m(Q_l^*)}{R_f}$	$m(Q_l^*)+m'(Q_l^*)Q_l^*\leq$ $\frac{R_f\left(1+C(s_l^*)\right)}{s_l^*}$	$m(0)>\frac{R_f\left(1+C(s_l^*)\right)}{s_l^*}$	(always $<$)	$e_p(Q_l^*)>\big[1-\frac{R_f\left(1+C(s_l^*)\right)}{s_l^*m(Q_s^*)}\big]^{-1}$
UL-JP	$Qp+R_f(W-Q(1+C(s)))(<Qd)$	$C'(s_j^*)=$ $\frac{m(Q_j^*)+R_f(W/Q-1-C(s_j^*))}{s_j^*R_f}$	$m(Q_j^*)+m'(Q_j^*)Q_j^*\leq$ $R_f\big(1+C(s_j^*)\big)$	$m(0)>R_f\big(1+C(s_j^*)\big)$	(can be $\lessgtr$)	$e_p(Q_j^*)>\big[1-\frac{R_f\left(1+C(s_j^*)\right)}{m(Q_s^*)}\big]^{-1}$
NL	0	$C'(s_0^*)=0$	$m(Q_0^*)+m'(Q_0^*)Q_0^*\leq R_f$	$m(0)>R_f$	(always $<$)	$e_p(Q_0^*)>\big[1-\frac{R_f}{m(Q_s^*)}\big]^{-1}$

$e_p\equiv-\frac{dI/I}{dp/p}=-\frac{m(Q)}{m'(Q)Q}$

Table 2: ***(Liability rules and solutions)*** *When the firm has market power and demand curve is not flat, full liability (FL) or unlimited liability with deep-pocket investors (UL) always results in first-best safety but lower quantity; the firm with limited liability has less incentives for safety but more incentives in quantity compared to FL. However, when the firm has unlimited liability, with small pockets (UL-JP) there can be under- and over-investment in both safety and quantity depending on the parameter values. With no liability (NL), the firm invests minimum in safety and there can be under- and over-investment in quantity depending on the demand elasticity.*

(1)	(2)	(3)	(4)
rule	$\Lambda(s,Q)$	FOC:(s)	FOC:(Q)
FB	-	$C'(s_s^*)=\frac{d}{R_f}$	$m(Q_s^*)\leq R_f\left(1+C(s_s^*)\right)$ $+(1-s_s^*)d$
FL/UL	Qd	$C'(s_u^*)=\frac{d}{R_f}$	$m(Q_u^*)+m'(Q_u^*)Q_u^*/N\leq$ $R_f\left(1+C(s_u^*)+k\right)+(1-s_u^*)d$
LL	$Qp+Qk(<Qd)$	$C'(s_l^*)=\frac{m(Q_l^*)+k}{R_f}$	$m(Q_l^*)+m'(Q_l^*)Q_l^*/N+k\leq$ $\frac{R_f\left(1+C(s_l^*)+k\right)}{s_l^*}$
UL-JP	$Qp+R_f(W/(N)-Q(1+C(s))-Qk)(<Qd)$	$C'(s_j^*)=$ $\frac{m(Q_j^*)+R_f(W/Q-1-C(s_j^*)-k)+k}{s_j^*R_f}$	$m(Q_j^*)+m'(Q_j^*)Q_j^*/N\leq$ $R_f\left(1+C(s_j^*)+k\right)$

red: adding number of firms.
blue: adding capital installment.

Table 3: ***(Increase competition and add capital)*** *When the number of firms N increases to infinity, equilibrium of full liability converges to first-best, whereas limiting liability (either because of the limited liability rule or the investors having small pockets) results in too high investment in quantity and too low investment in safety and is socially inefficient.*
Requiring purchased capital as an example of general capital requirement may push the equilibria back to first-best for limited liability, but either do not change incentives for small pockets under unlimited liability or tend to be inefficient when the investors are deep-pocket.